Connections between discrete and continuous optimization

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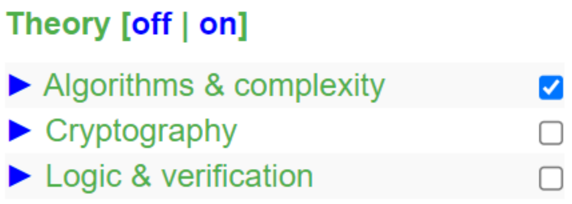
“CS Theory”

- “Theoretical Computer Science” (TCS) / “Theory of Computing” (ToC)

- What do we do?
  - We prove theorems about computation

STOC/FOCS/SODA community



csrankings.org

- Sometimes called "algorithms and complexity"
  - STOC (**S**ymp. on **T**heory **of** **C**omputing, since 1969)
  - FOCS (symp on **F**oundations **of** **C**omputer **S**cience, since 1960)
  - SODA (**S**ymp. **on** **D**iscrete **A**lgorithms, since 1990)

- Other significant communities of TCS
  - Logic and theory of programming languages
  - Optimization/numerical analysis
  - Information/coding/control/systems performance theory

Influence from Mathematics

- Everything you claim should be proved.
- Authors are ordered alphabetically.
- Some papers are published in math journals
- Avi Wigderson and László Lovász won 2021 Abel Prize.
  - Avi’s PhD is from Computer Science!

A “typical paper” looks like…

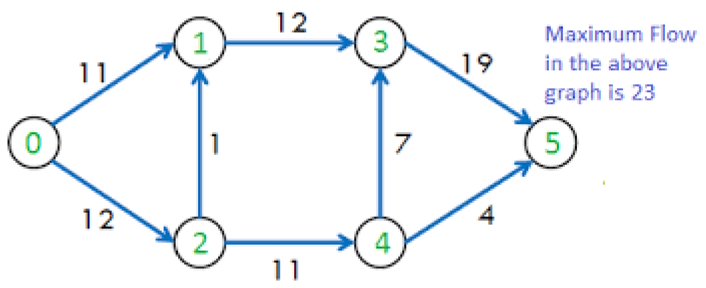
- “Task/problem” that you want to do with computers
  - Finding shortest path, factoring integer, finding optimal parameters for neural nets
  - Any well-defined “function” specifying (valid input) => (desired output)

- “Model of computation”
  - (poly-time/randomized/non-deterministic) Turing machine
  - streaming/online/dynamic
  - distributed/parallel
  - quantum
  - crypto/communication
  - market/brain/evolution (computational lens)

- Can do it (there’s an algorithm) or cannot do it (there’s no algorithm)!

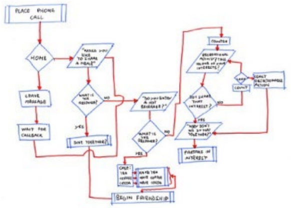
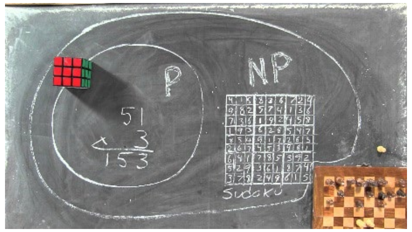
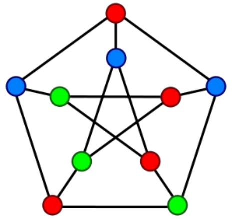
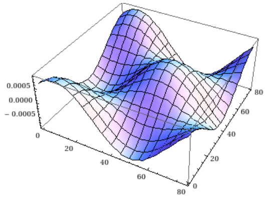
Some cool recent things

- Computing Maximum Flow of graph $G = (V, E)$
  - [Ford-Fulkerson 56] $O(|E|f)$ where f is max flow --- not poly-time
  - [Dinitz 70, Edmonds-Karp 72] $O(|V|^2|E|)$
  - …
  - [CKLPPS 22] $O(|E|^{1.0001})$
    - Combination of interior point method (continuous) + graph theory (discrete)



- Traveling Salesperson Problem (TSP)
  - Given $G = (V, E)$, find shortest tour that visits every vertex at least once.
  - [Christofides 76] 1.5-approximation
  - [KKO 21] $(1.5 - 10^{-36})$-approximation!
    - Combination of graph cut representation (discrete) + real-stable polynomials (continuous).

Plan

Abstraction / Objects	Algorithms	Complexity
		
Discrete 		
Continuous 		

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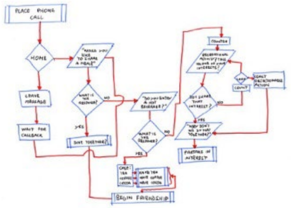
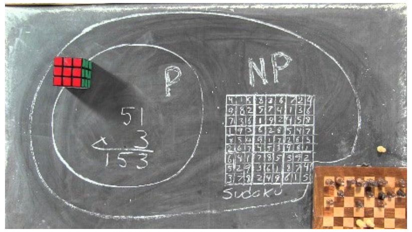
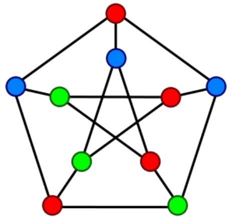
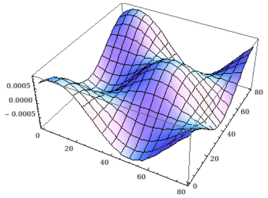
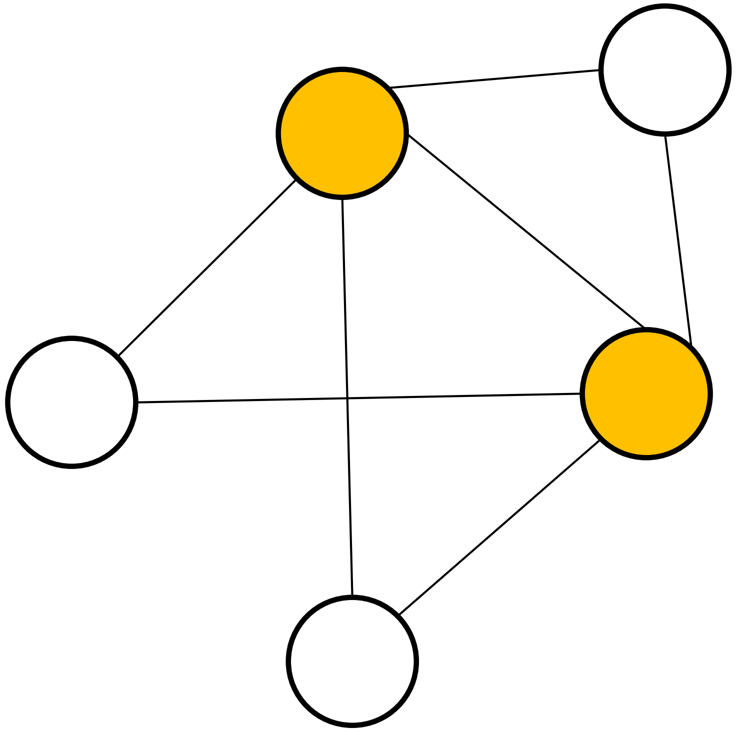
Abstraction / Objects	Algorithms 	Complexity 
Discrete 		
Continuous 		

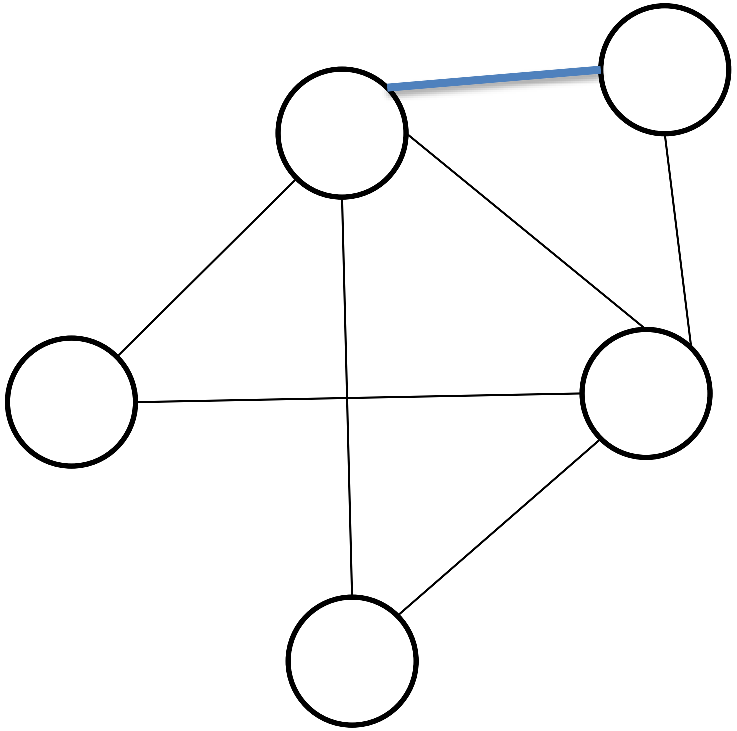
Example 1: Vertex Cover

- Minimization
- Input
  - (Undir.) Graph G = (V, E)
- Output
  - Subset U of V such that
  - U intersects (covers) every edge!
- Objective Function
  - Cardinality of U



Example 1: Vertex Cover

- Choose an edge in G
- Add both endpoints to U.
- Delete these two vertices from G,
  - Including all edges incident on them
- Repeat until G has no edge.



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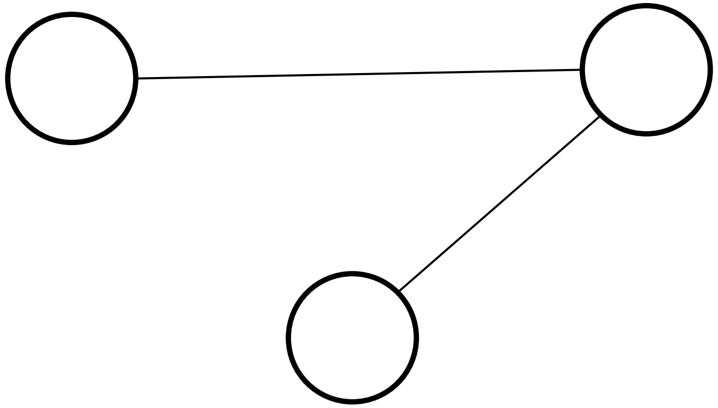

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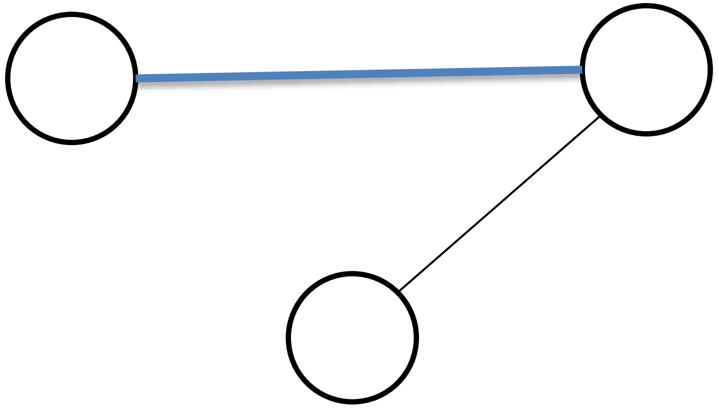

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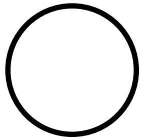

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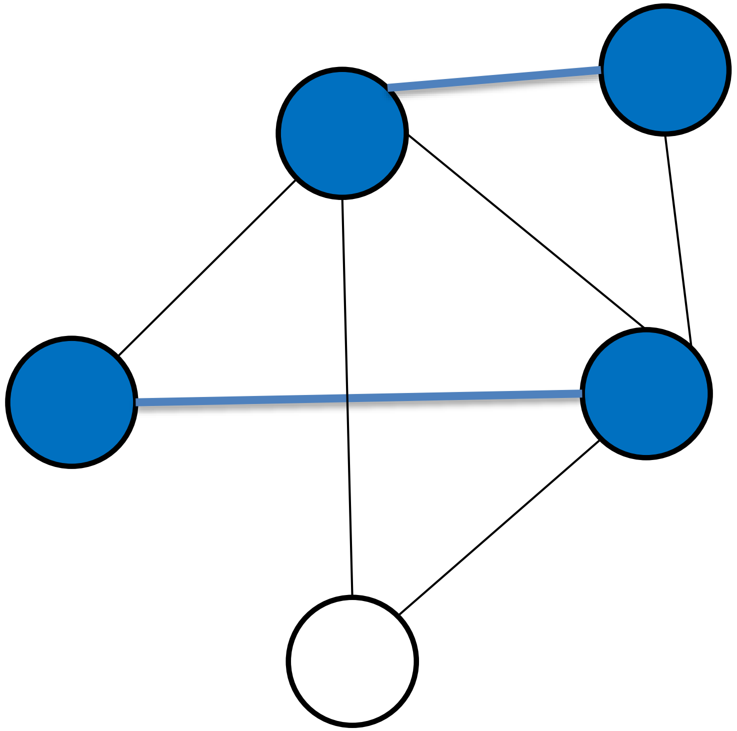
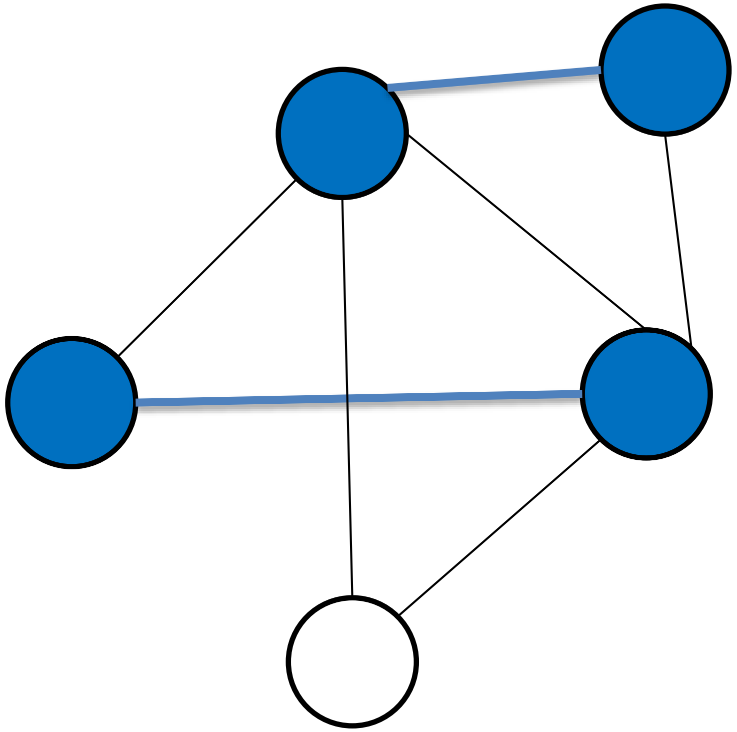


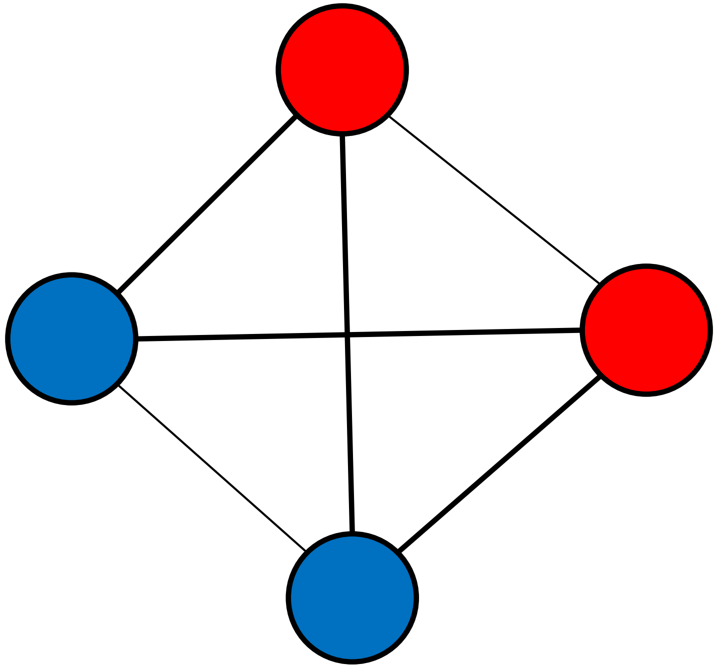
Example 1: Vertex Cover

- Let p = # of blue (chosen) edges
  - $|U| = 2p$
  - Every Vertex Cover has to contain at least one vertex from each blue edge!
  - All blue edges do not share a vertex
  - $OPT \geq p$.
  - Therefore, $|U| \leq 2OPT$.



Example 2: Max Cut

- Maximization
- Input
  - Graph G = (V, E)
- Output
  - Coloring V → {B, R}
- Objective Function
  - # of edges between Blue and Red



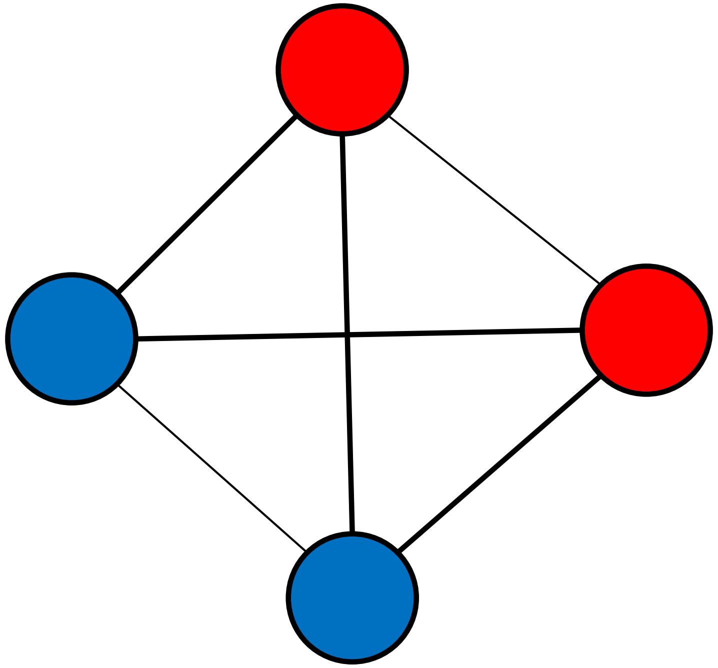
Example 2: Max Cut

- For each v, randomly (and independently) color B or R.

- Each edge is (B-R) with probability 0.5.
  - 0.5-approximation

- [GW94] Semidefinite programming

$$\min_{0 \le x \le 1} \frac{1}{\pi} \arccos(1 - 2x) \approx 0.878$$



Big Open Questions

- Can we do better?
  - 1.99-approximation for Vertex Cover
  - 0.879 for Max-Cut

Probably not!

	Vertex Cover	Max Cut
Algorithm	**2**	**0.878 (GW 94)**
NP-Hardness	**1.36 (DS 05)**	**0.941(TSSW01)**
UG-Hardness	**2 (KR 08)**	**0.878(KKMO07)**

- [Khot 02] Unique Games Conjecture (UGC).
- [KR08, KKMO07] Tight hardness for Vertex Cover and Max-Cut assuming UGC

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	Vertex Cover	Max Cut
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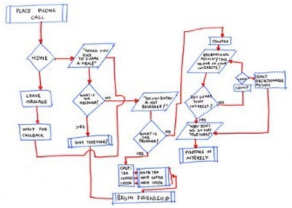
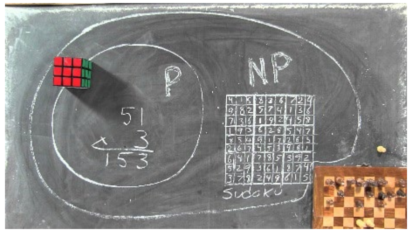
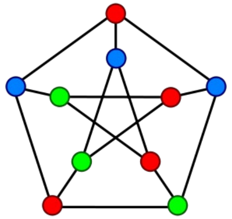
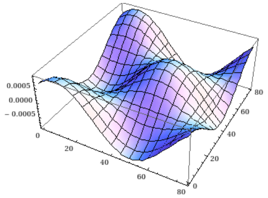
		
Discrete 	Vertex Cover Max Cut	
Continuous 		

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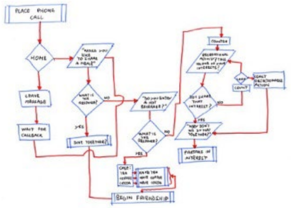
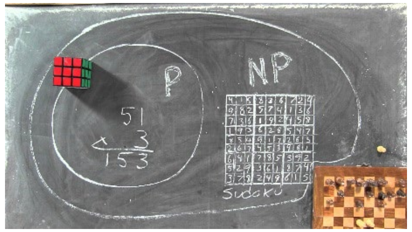
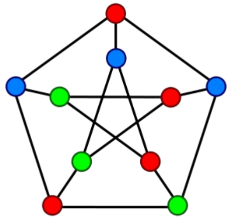
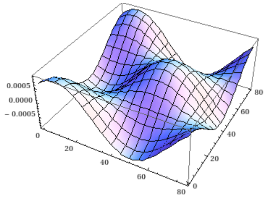
		
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 		

Small Set Expansion

- [RS 10] Small Set Expansion Hypothesis (SSEH)
  - SSEH $\Rightarrow$ UGC $\Rightarrow$ (optimal hardness for …)
  - Don’t know whether SSEH $\Leftarrow$ UGC

- Will focus on SSEH..

Spectral Graph Theory 101

- Let $G = (V, E)$ be a d-regular graph. $(n = |V|)$
- Given $S \subseteq V$,
  - $\Phi(S) := \frac{|E(S, V \setminus S)|}{d|S|}$
  - Note that $0 \leq \Phi(S) \leq 1$
  - S is "expanding" when $\Phi(S) \approx 1$.
  - Fair to consider S with $|S| \leq n/2$.
- $\Phi_G = \min_{|S| \leq n/2} \Phi(S)$
  - Expansion of "the least expanding set"

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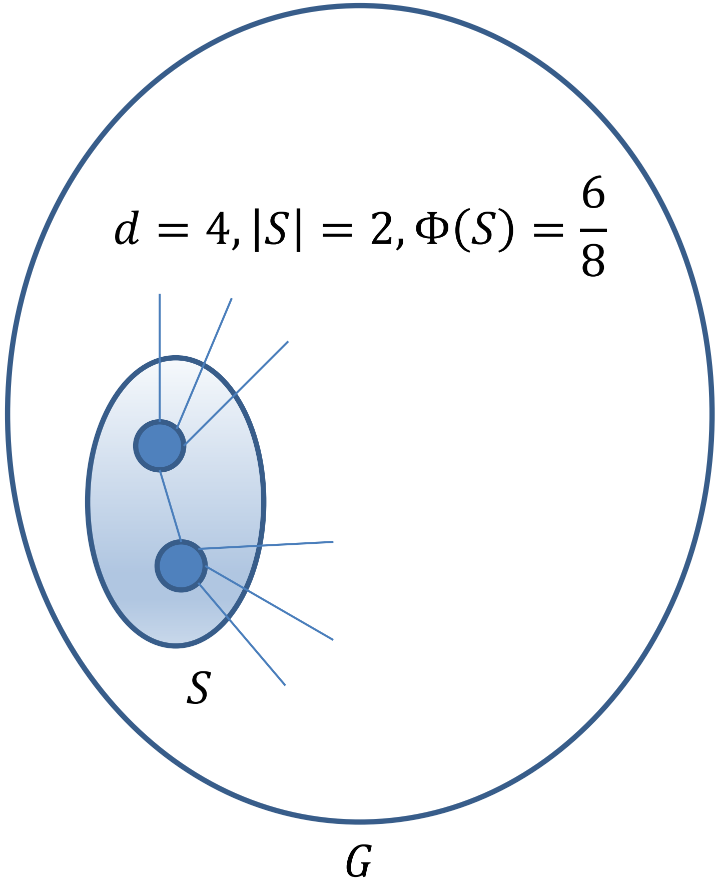

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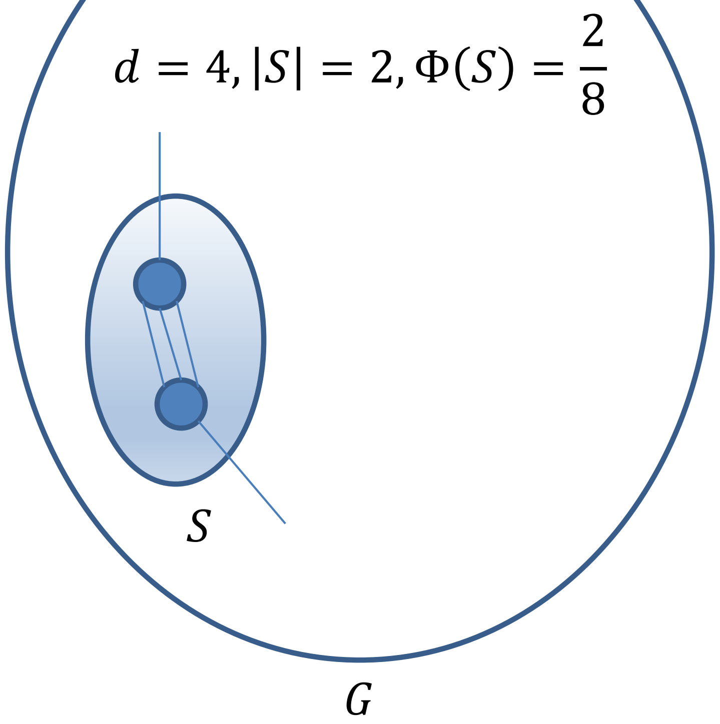
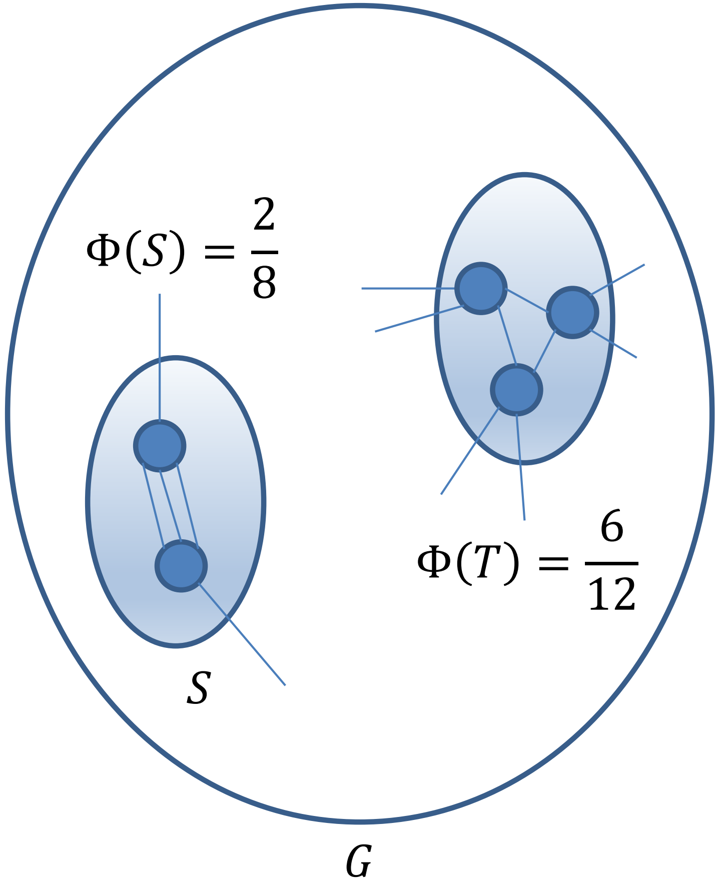


$\Phi(S) = \frac{|E(S, V\setminus S)|}{d|S|}$, $\Phi_G = \min_{|S|\leq n/2} \Phi(S)$

SGT 101

- "Expansion problem"
  - Given G with $\Phi_G \leq \epsilon$,
  - Find T s.t.
    - $|T| \leq n/2$
    - $\Phi(T) \leq \epsilon'$
  - Want $\epsilon' \to 0$ as $\epsilon \to 0$.

- [Cheeger's inequality] Can solve "Expansion problem" with $\epsilon' = \sqrt{\epsilon}$



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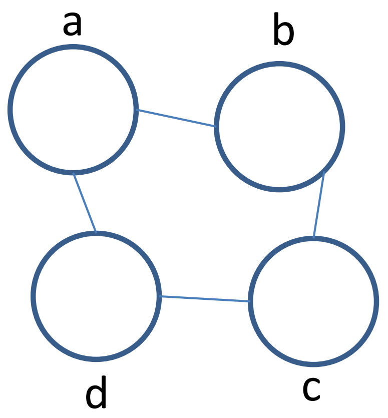
[Cheeger] Given G with $\Phi_G = \epsilon$, can find S with $\Phi(S) \leq \sqrt{\epsilon}$

SGT 101

- Given d-regular G, consider normalized adjacency matrix A.
  - $A_{i,j} = 1/d$ if $(i,j) \in E$.
- Let $\lambda_1 \geq \cdots \geq \lambda_n$ eigenvalues of A
  - $\lambda_1 = 1$ ($Ax = x$ when $x = (1,1,\ldots,1)$).



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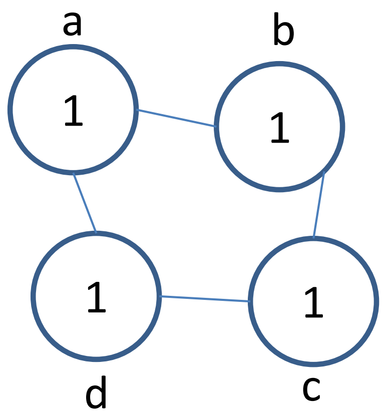

	a	b	c	d
a	0	½	0	½
b	½	0	½	0
c	0	½	0	½
d	½	0	½	0

A

x
1
1
1
1

=

y
1
1
1
1

$\lambda_1 = 1,$

$\Phi(S) = \frac{|E(S, V\setminus S)|}{d|S|}$, $\Phi_G = \min_{|S| \le n/2} \Phi(S)$

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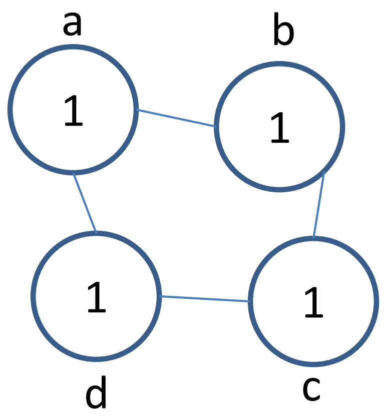
  - $A_{i,j} = 1/d$ if $(i,j) \in E$.
- Let $\lambda_1 \ge \cdots \ge \lambda_n$ eigenvalues of A
  - $(Ax)_i = \frac{1}{d} \sum_{(i,j) \in E} x_j$
    - "Average x-value of nbrs"



A:

	a	b	c	d
a	0	½	0	½
b	½	0	½	0
c	0	½	0	½
d	½	0	½	0

x =

1
1
1
1

= y

1
1
1
1

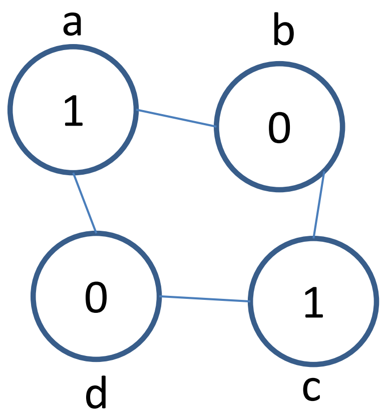
$\lambda_1 = 1,$

$\Phi(S) = \frac{|E(S, V \setminus S)|}{d|S|}$, $\Phi_G = \min_{|S| \le n/2} \Phi(S)$

[Cheeger] Given G with $\Phi_G = \epsilon$, can find S with $\Phi(S) \le \sqrt{\epsilon}$

SGT 101 - Intuition

- $(Ax)_i = \frac{1}{d} \sum_{(i,j) \in E} x_j$
  - "Average value of nbrs"
- If x is indicator vector of "expanding set" S,
  - Ax will have not much values in S
  - x and Ax are "far"



$\Phi(\{a, c\}) = 1$

A:

	a	b	c	d
a	0	½	0	½
b	½	0	½	0
c	0	½	0	½
d	½	0	½	0

x
1
0
1
0

=

y
0
1
0
1

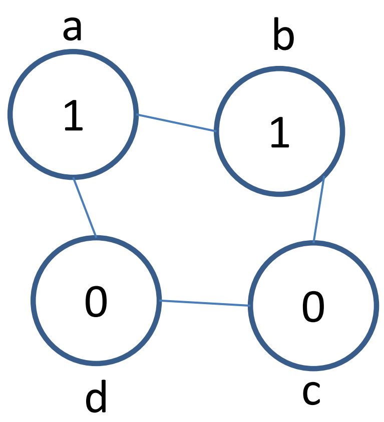
SGT 101 - Intuition

$\Phi(S) = \frac{|E(S,V\setminus S)|}{d|S|}$, $\Phi_G = \min_{|S|\leq n/2} \Phi(S)$

[Cheeger] Given G with $\Phi_G = \epsilon$, can find S with $\Phi(S) \leq \sqrt{\epsilon}$

- If x is indicator vector of "non-expanding set" S,
  - Ax will have some values in S
  - x and Ax are "close"
    - Large eigenvalue and its eigenvector!

- [Cheeger] Converse of above
  - If $x \approx Ax$ and far from $(1, \ldots, 1)$,
    - E.g., $\lambda_2 x = Ax$,
  - x is "close" to indicator vector of non-expanding small set S.



$\Phi(\{a, b\}) = 1/2$

	a	b	c	d	x		y
a	0	½	0	½	1		½
b	½	0	½	0	1	=	½
c	0	½	0	½	0		½
d	½	0	½	0	0		½

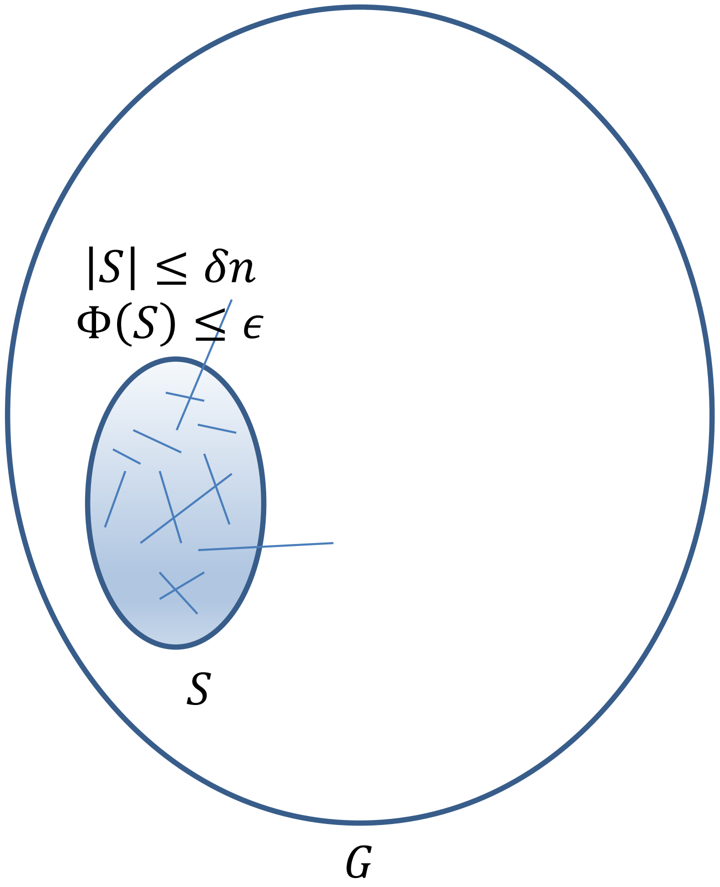
A x y

Small Set Expansion

$\Phi(S) = \frac{|E(S,V\setminus S)|}{d|S|}$, $\Phi_G = \min_{|S|\le n/2} \Phi(S)$

[Cheeger] Given G with $\Phi_G = \epsilon$, can find S with $\Phi(S) \le \sqrt{\epsilon}$

- $\Phi_{\delta,G} = \min_{|S|\le\delta n} \Phi(S)$.
  - So $\Phi_G = \Phi_{1/2,G}$.
- Q] Is there Cheeger-type algorithm?
  - Given $\Phi_{\delta,G} = \epsilon$, find T with
  - $|\mathrm{T}| \le \delta n, \Phi(T) \le \epsilon'$.
- [RS10] (SSE Hypothesis)
  - $\forall\epsilon > 0$, $\exists\delta > 0$ s.t. given G with $\Phi_{\delta,G} = \epsilon$,
  - NP-hard to find T with $|T| \le \delta n$, $\Phi(T) \le 1 - \epsilon$



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[Cheeger] Given G with $\Phi_G = \epsilon$, can find S with $\Phi(S) \le \sqrt{\epsilon}$

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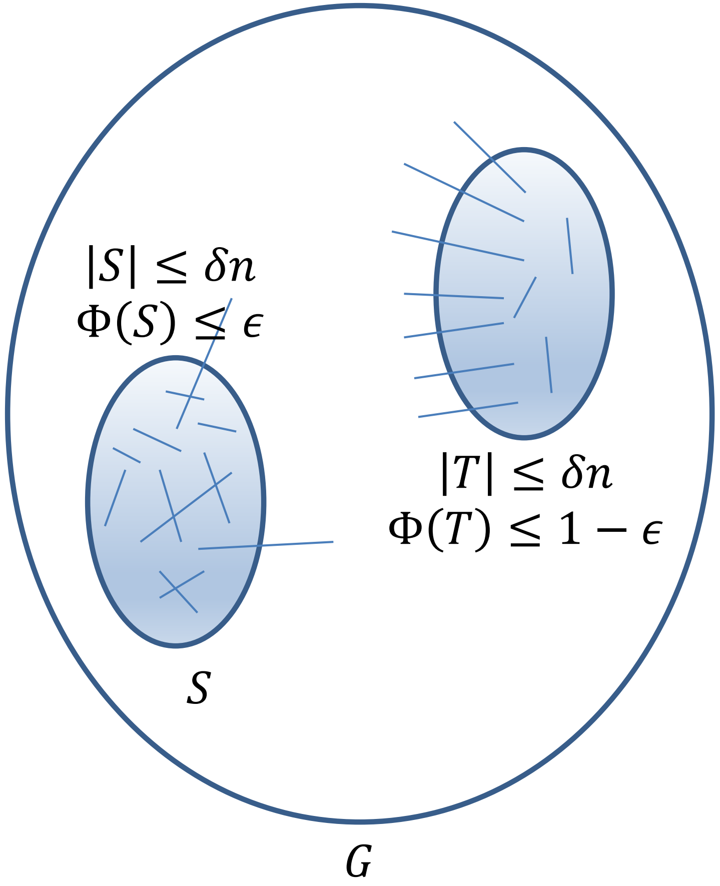


$$\Phi_{\delta,G} = \min_{|S| \leq \delta n} \Phi(S)$$

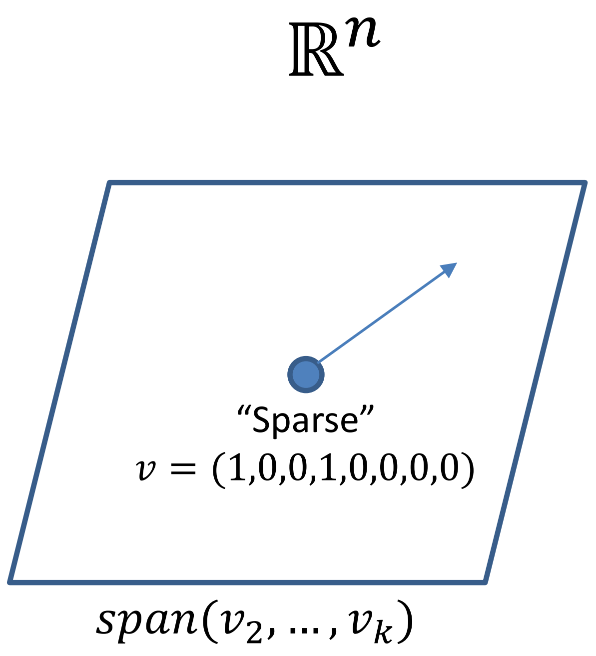
[SSEH] Given G with $\Phi_{G,\delta} = \epsilon$, ***cannot*** find S with $\Phi(S) \leq 1 - \epsilon$.

Small Set Expansion

- Try previous algorithm.
  - Recall $1 = \lambda_1 \geq \cdots \geq \lambda_n$ with eigenvectors $v_1, \ldots, v_n$
  - Take v_2 s.t. $Av_2 = \lambda_2 v_2$.
- If v_2 is ***sparse indicator***
  - E.g., indicator vector of some set $|S| \leq \delta n$.
  - S is a small non-expanding set!
- Even if $span(v_2, \ldots, v_k)$ contains a sparse indicator vector for small k, we are good.

Finding “Sparse” Vector

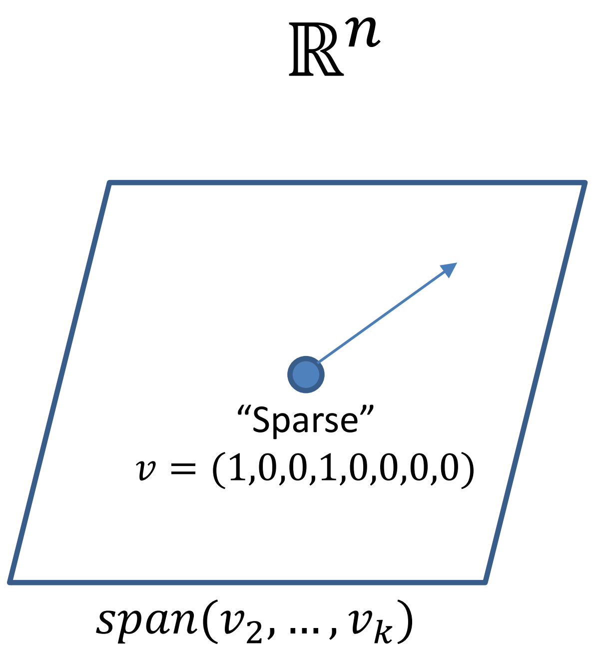
- Now if want to find a “sparse” vector in linear space $span(v_2, \dots, v_k)$.



Finding "Sparse" Vector

- Now if want to find a "sparse" vector in linear space $V \subseteq \mathbb{R}^n$.

- What is good notion of "sparsity"?
  - # of nonzero entries is too susceptible to noise.



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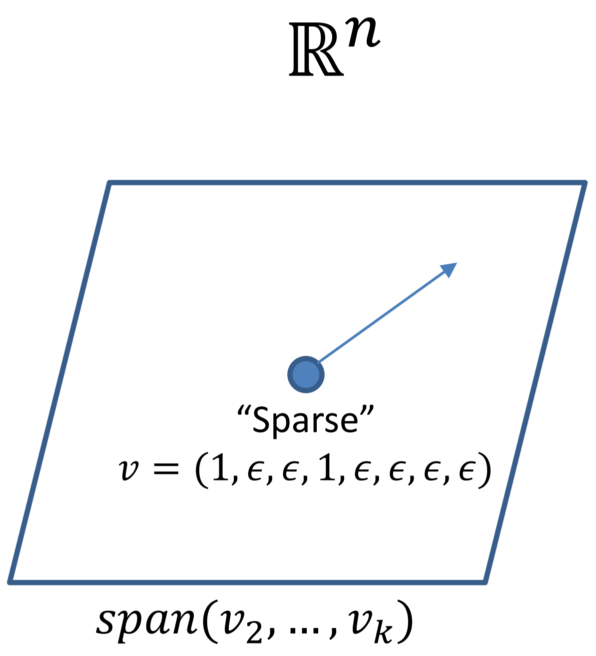
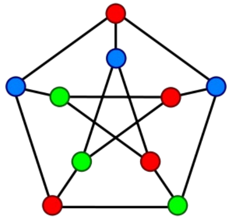
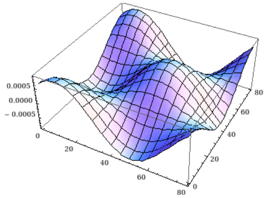
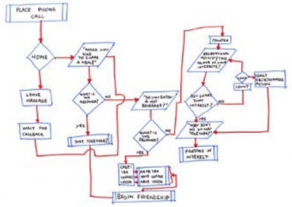
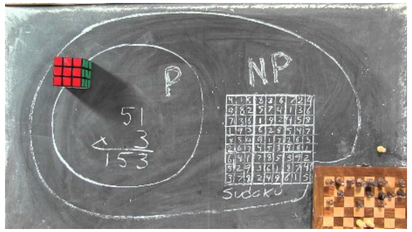


Plan

Abstraction / Objects	Algorithms	Complexity
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 		

Algorithms: 

Complexity: 

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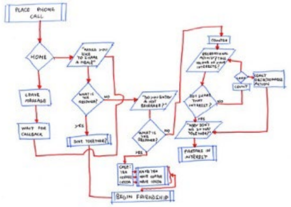
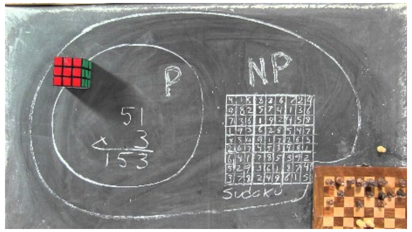
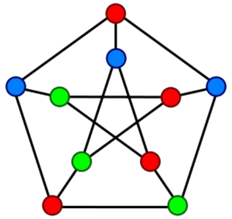
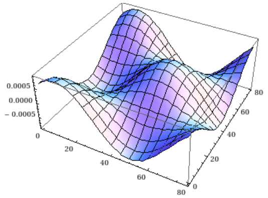
		
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 		Operator Norms

Norms

- Given a vector $x = (x_1, \dots, x_n)$ and $p \geq 1$,
  - $||x||_p := (\sum_{i=1}^{n} |x_i|^p)^{1/p}$.
  - $||x||_\infty := \max_i |x_i|$.
- Facts
  - $||x||_p \geq ||x||_q$ if $p \leq q$
  - For $q > p$, $||x||_q / ||x||_p$ is maximized when x has only one nonzero entry.
  - For $q < p$, $||x||_q / ||x||_p$ is maximized when it is *well-spread* (i.e., $|x_1| = |x_2| = \cdots = |x_n|$)

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1	0	0	0	0

$$||x||_1 = 1, ||x||_2 = 1, ||x||_4 = 1$$

$$\frac{||x||_1}{||x||_2} = 1, \qquad \frac{||x||_4}{||x||_2} = 1$$

$1/\sqrt{5}$	$1/\sqrt{5}$	$1/\sqrt{5}$	$1/\sqrt{5}$	$1/\sqrt{5}$

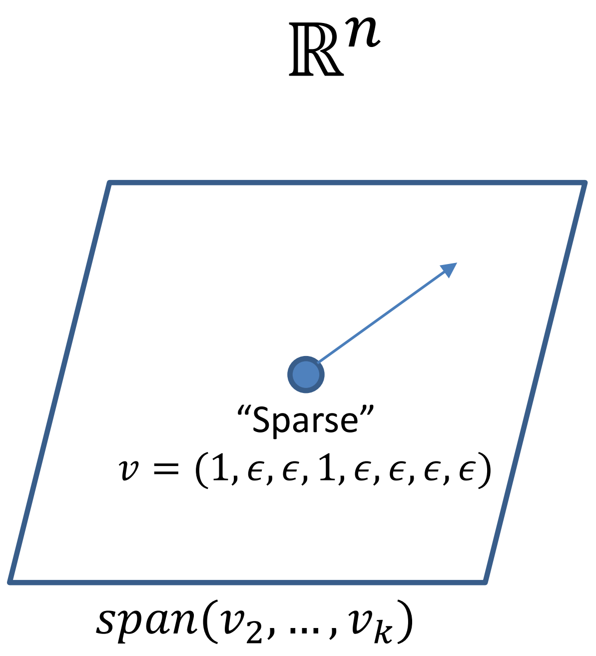
$$||x||_1 = \sqrt{5}, ||x||_2 = 1, ||x||_4 = 1/\sqrt[4]{5}$$

$$\frac{||x||_1}{||x||_2} = \sqrt{5} = 2.24, \frac{||x||_4}{||x||_2} = \frac{1}{\sqrt[4]{5}} = 0.66$$

Small Set Expansion

- Now if want to find a "sparse" vector in linear space $V \subseteq \mathbb{R}^n$.

- What is good notion of "sparsity"?
  - # of nonzero entries is too susceptible to noise.



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- Now if want to find a "sparse" vector in linear space $V \subseteq \mathbb{R}^n$.
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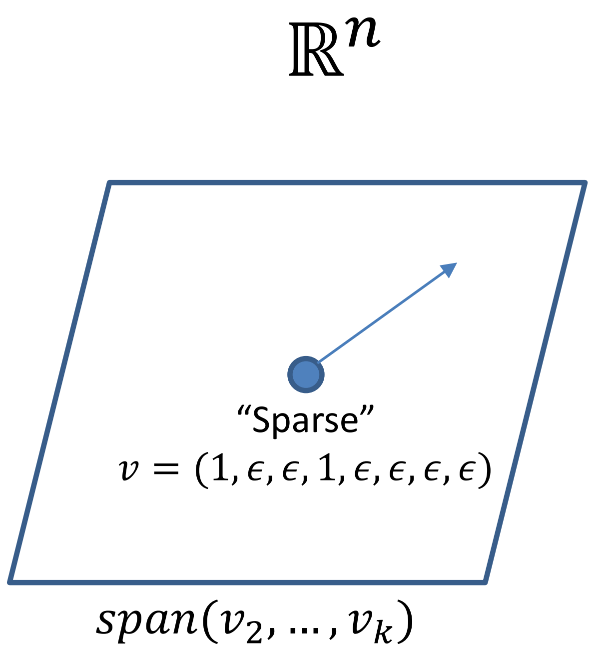
  - $||x||_q/||x||_p$ is large when $q > p$.



$||v||_q/||v||_p$ must be large!

Small Set Expansion

- Now if want to find a "sparse" vector in linear space $V \subseteq \mathbb{R}^n$.
- What is good notion of "sparsity"?
  - # of nonzero entries is too susceptible to noise.
  - $||x||_q/||x||_p$ is large when $q > p$.
- $(p = 2)$ Let A be matrix whose columns form orthonormal basis of V.
  - $\forall x: ||Ax||_2 = ||x||_2$.
  - $Ax \in V$
- So, finding a "sparse vector" in V became finding x that maximizes

$$\frac{||Ax||_q}{||Ax||_2} = \frac{||Ax||_q}{||x||_2}$$

Operator Norms

- For matrix A, and p, q,

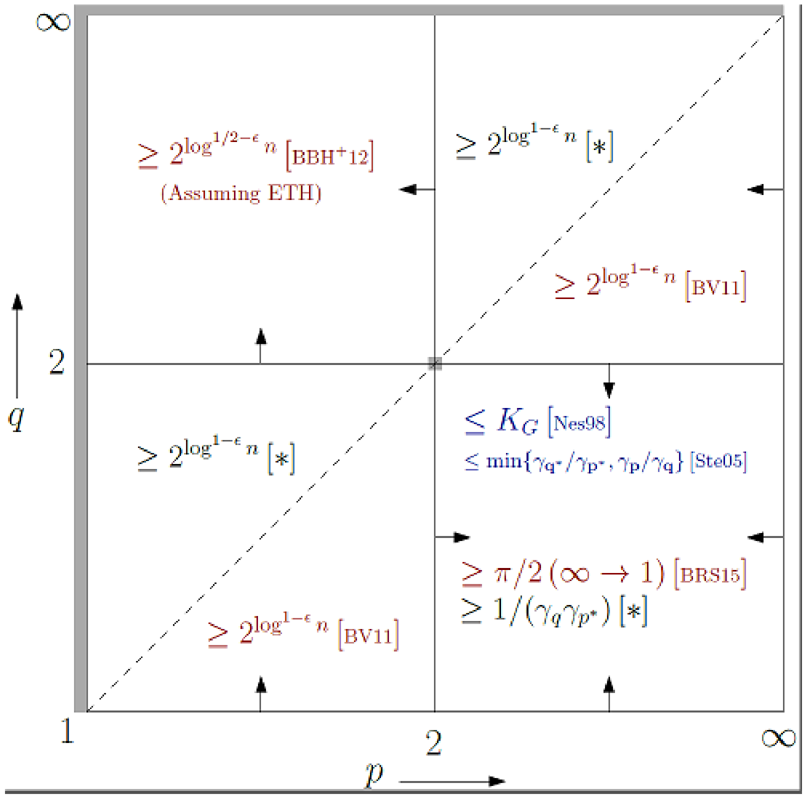
$$||A||_{p\to q} := \max_{x\neq 0} \frac{||Ax||_q}{||x||_p} = \max_{x:||x||_p=1} ||Ax||_q$$

  – Can we (approximately) compute it?

- Connections to machine learning, quantum computing, etc.
  – When q is even integer, $||Ax||_q^q$ is degree-q polynomial in x!
- [BBHKSZ 12] When $p = 2, q > 2$, a good (constant) approx. algorithm to compute $||A||_{p\to q}$ solves Small Set Expansion.

Approximating $||A||_{p \to q}$

- $p = q = 2$: spectral norm.
- $p \geq q$: well-understood.
  - $p \geq 2 \geq q$: K_G-approx. ($1.67 \leq K_G \leq 1.79$)
  - Otherwise: No c-approx. is possible for any $c > 1$.
- [BGGLT 19] If $2 < p < q$ or $p < q < 2$,
  - No c-approx. is possible for any $c > 1$.
  - First NP-hardness for $p < q$
  - Don't cover $p = 2, q > 2$.



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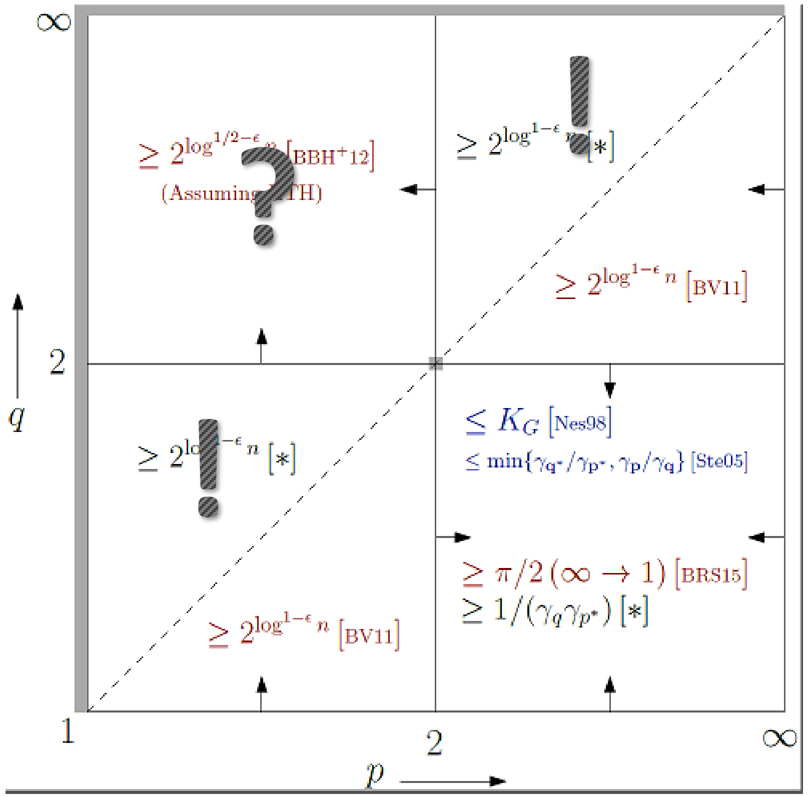
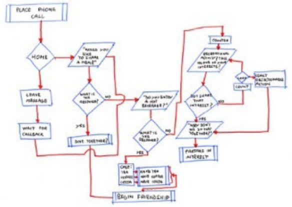
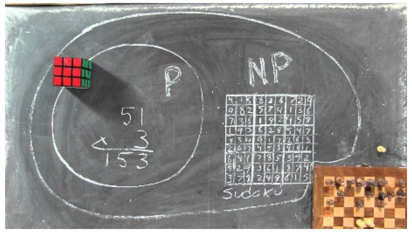
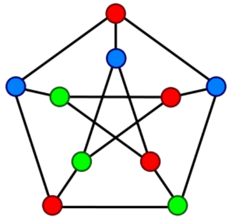
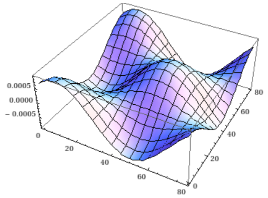


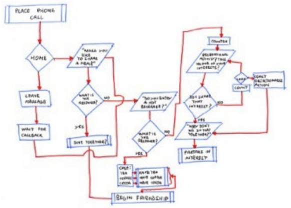
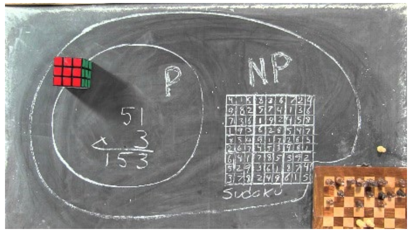
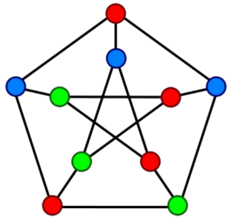
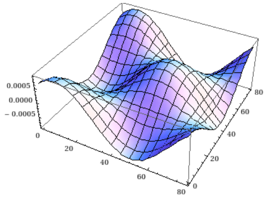
One-line intuitions for proofs

- $p \geq q$: Closer to "discrete problems"!
  - When $p = \infty$, $||x||_\infty \leq 1$ means $x_i \in [-1, +1]$ for every i
    - WLOG, can even assume $x_i \in \{-1, +1\}$
  - Tools from discrete problems work when $p \geq q$.
- [BGG**L**T 19] Hardness for $2 < p < q$
  - It is hard to find x with $||x||_p = 1$ with large $||Ax||_2$.
  - Dvoretzky's theorem: If B is a "random matrix", then $||By||_q \approx ||y||_2$
  - It is hard to find x with $||x||_p = 1$ with large $||BAx||_q$!

Plan

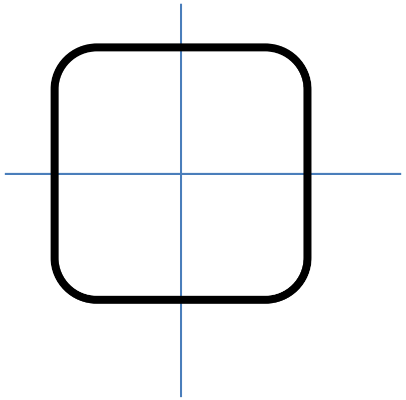
Abstraction Objects	Algorithms 	Complexity 
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 		Operator Norms

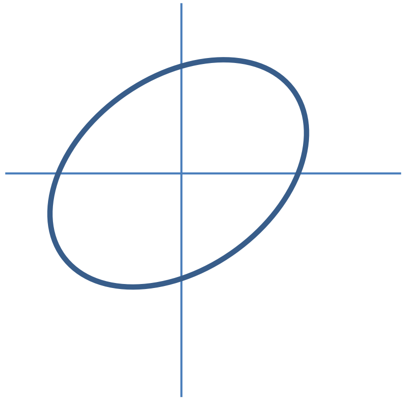
Plan

Abstraction / Objects	Algorithms 	Complexity 
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 	Quadratic Maximization	Operator Norms

Quadratic optimization over general norm

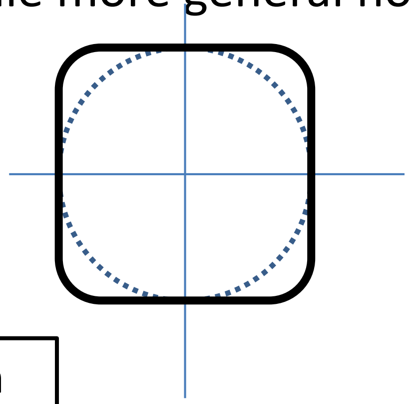
- $||A||_{p\to 2}$: Easy when $p \geq 2$ (K_G-approx. $1.67 \leq K_G \leq 1.79$) but (believed to be) hard when $p < 2$.
- Can we generalize "p"?
- Let $B \subseteq \mathbb{R}^n$ be a symmetric ($B = -B$) convex set.
  - Defines a general "norm": $||x||_B$ = (smallest $t > 0$ s.t. $x/t \in B$)
  - Example: Unit ℓ_p ball $\{x: ||x||_p \leq 1\}$.
- Input: $m \times n$-matrix A.
- Output: $x \in B$ that maximizes $||Ax||_2$.
- Which B allows $O(1)$-approximation?





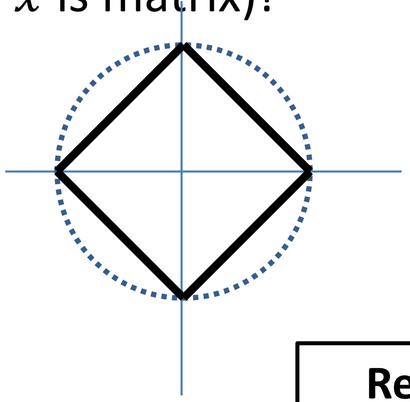
Quadratic optimization over general norm

- Let $B \subseteq \mathbb{R}^n$ be a symmetric ($B = -B$) convex set.
- Input: $m \times n$-matrix A.
- Output: $x \in B$ that maximizes $||Ax||_2$.
- Which B allows $O(1)$-approximation?
- Answer [B**L**N 21]: When B is "fatter" than ℓ_2 ball!
  - Using notion of "type" from functional analysis
  - Can handle more general norms (e.g., when x is matrix)!



Tools from discrete optimization

YES



NO

Reduction from ℓ_p with p<2

Plan

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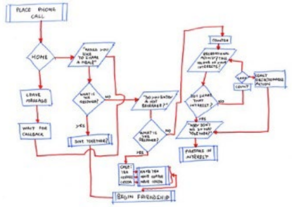
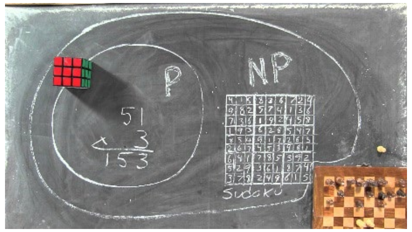
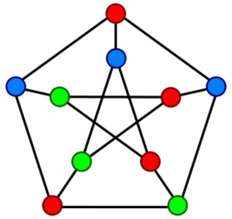
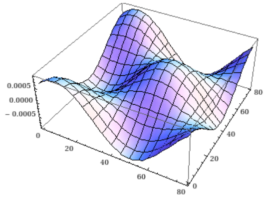
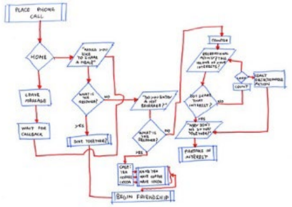
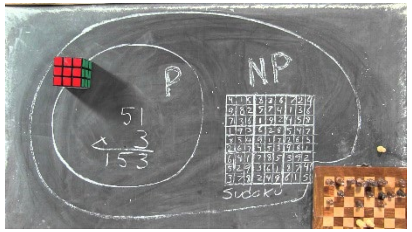
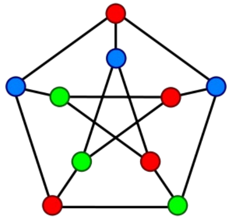
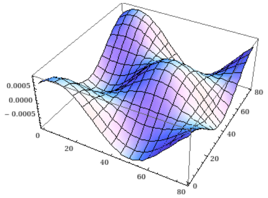
		
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 	Quadratic Maximization	Operator Norms

Conclusion

- Continuous and Discrete optimization
  - This talk: through operator norms
  - Other connections (e.g., Max-flow, TSP)
- (Approximately) finding global optimum for non-convex functions
  - Many viewpoints: Statistics, Optimization, Machine learning, Physics, Pure math, CS, etc.
  - Important to build “bridges”.
    - Recently started to talk to each other..
  - Exciting time to study!

Thank you!

Abstraction / Objects	Algorithms	Complexity
		
Discrete 	Vertex Cover Max Cut	Unique Games Small-Set Expansion
Continuous 	Quadratic Maximization	Operator Norms