High level abstractions for irregular computations

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1. INTRODUCTION: THE PROBLEM

Programming for performance

- Is laborious
  - Particularly so when the target machine is parallel.
    - Must decide what to execute in parallel
    - In what order to traverse the data
    - How to partition the data …
- Today, tuning must be done manually.
- Maintenance is difficult
  - and the result is not portable.

Programming for performance

- There is still much room for progress in high performance programming methodology.
- Our goal is to facilitate (automate) tuning and enable machine independent programming.

2. IRREGULAR COMPUTATIONS

What are irregular computations ?

- Irregular computations are difficult to define.
- One possible definition is:
  - Computations on “irregular” data structures.
  - Irregular data structures = not dense arrays.
    - Pingali et al. *The tao of parallelism in algorithms*. PLDI’11
  - And subscript expressions non-linear.

What are irregular computations ?

- Examples include
  - Subscripted subscripts:
    - `A[C[i]]`
  - Linked lists traversal:
    - `leftCell = leftCell->ColNext;`
    - `result+=leftCell->Value;`

Programming irregular computations for performance

- Today, programming of irregular computations is typically at a low level of abstraction.

```
do diag=1,Nj, 2    ! two-way unroll amd Jam
      diaglen=min(   ( jd_ptr(diag+1)-jd_ptr(diag) ) , \
                     ( jd_ptr(diag+2)-jd+ptr(diag+1) )    )
      offset1  = jd_ptr(diag)  - 1
      offset2  = jd_ptr(diag+1) - 1
      do i=1, diagLen
           C(i) = C(i) + val(offset1+i)*B(col_idx(offsetr1+i))
           C(i) = C(i) + val(offset2+i)*B(col_idx(offset2+i))
      end do
      ! peeled off iterations
      offset1 = jd_ptr(diag)
      do i=(diagLen+1), (jd_ptr(diag+1)-jd_ptr(diag))
           C(i) = C(i)+val(offset1+i)*B(col_idx(offswt1+i))
      end do
end do
```

G. Hager and G. Wellein. Introduction to High Performance Computing for Scientists and Engineers.CRC Press

Programming irregular computations for performance

- Programming and optimizing irregular computations more challenging than for linear-subscripts computations.

- Want to have compiler support for programming at a low level of abstraction to enable
  - Automatically mapping computation onto diverse machine classes
  - Locality enhancement
  - Other compiler optimizations (e.g. common subexpression eliminations)
- In addition, often using higher levels of abstractions and applying optimizations at that level is a better solution.

- Both approaches will be discussed in this presentation

When computations are not irregular

- Compiler can do a good job at transforming programs:
  - Loop interchange
  - Loop distribution
  - Tiling
  - …
- And we have the the technology to generate efficient code for a variety of platforms even when the initial code is sequential
  - Vector extensions
  - Multicores
  - Distributed memory machines
  - GPUs

Example
Automatic transformation of a regular computation

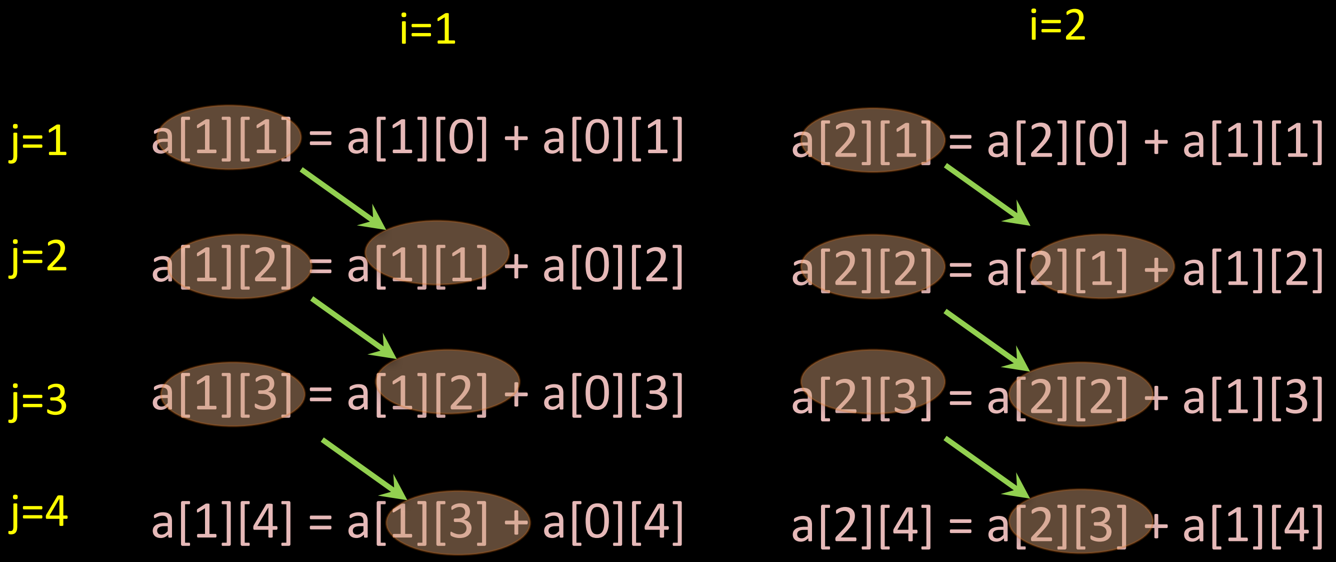
- Consider the loop

```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```

Example
Automatic transformation of a regular computation

- Compute dependences (part 1)

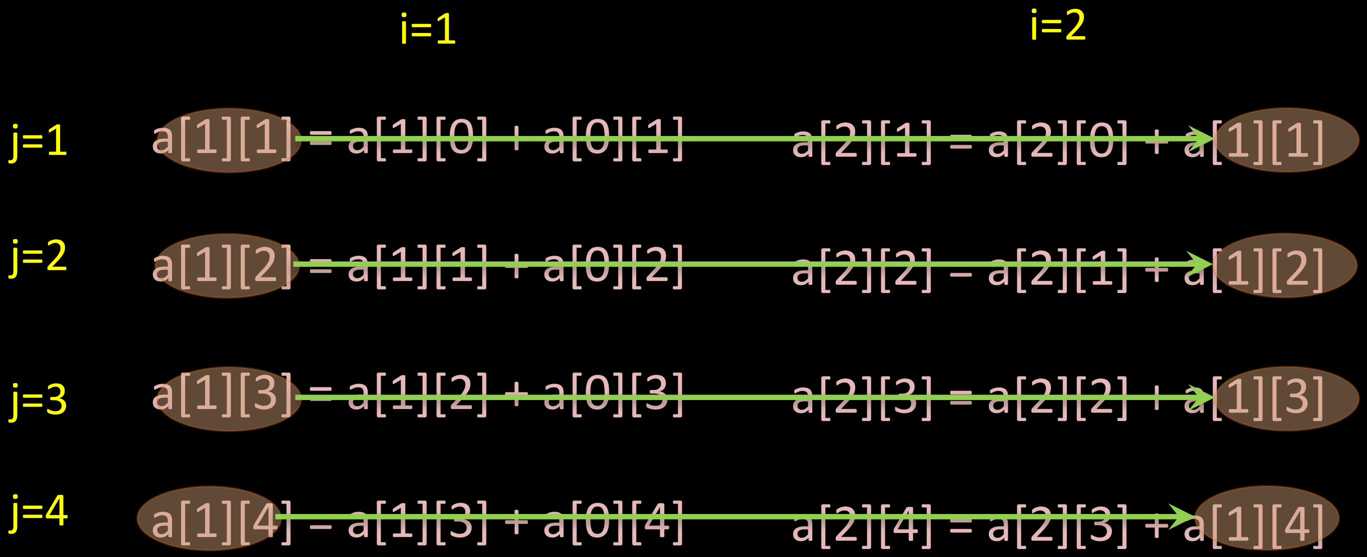
```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```


i=1
i=2
j=1
j=2
j=3
j=4
a[1][1] = a[1][0] + a[0][1]
a[1][2] = a[1][1] + a[0][2]
a[1][3] = a[1][2] + a[0][3]
a[1][4] = a[1][3] + a[0][4]
a[2][1] = a[2][0] + a[1][1]
a[2][2] = a[2][1] + a[1][2]
a[2][3] = a[2][2] + a[1][3]
a[2][4] = a[2][3] + a[1][4]

Example
Automatic transformation of a regular computation

- Compute dependences (part 2)

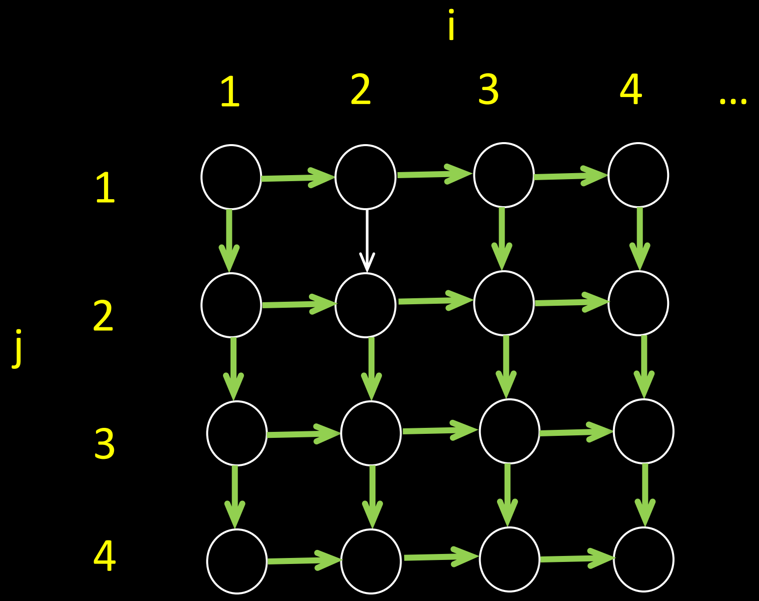
```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```


i=1
i=2
j=1
a[1][1] = a[1][0] + a[0][1]
a[2][1] = a[2][0] + a[1][1]
j=2
a[1][2] = a[1][1] + a[0][2]
a[2][2] = a[2][1] + a[1][2]
j=3
a[1][3] = a[1][2] + a[0][3]
a[2][3] = a[2][2] + a[1][3]
j=4
a[1][4] = a[1][3] + a[0][4]
a[2][4] = a[2][3] + a[1][4]

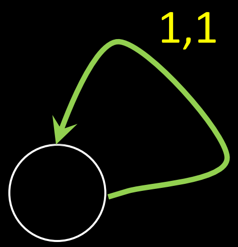
Example

Automatic transformation of a regular computation

```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```



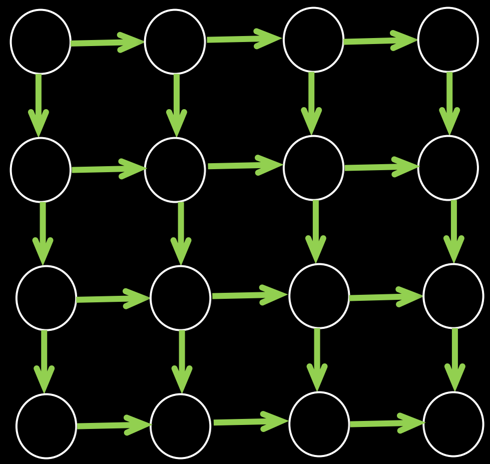
or



Example
Automatic transformation of a regular computation

- Find parallelism

```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```

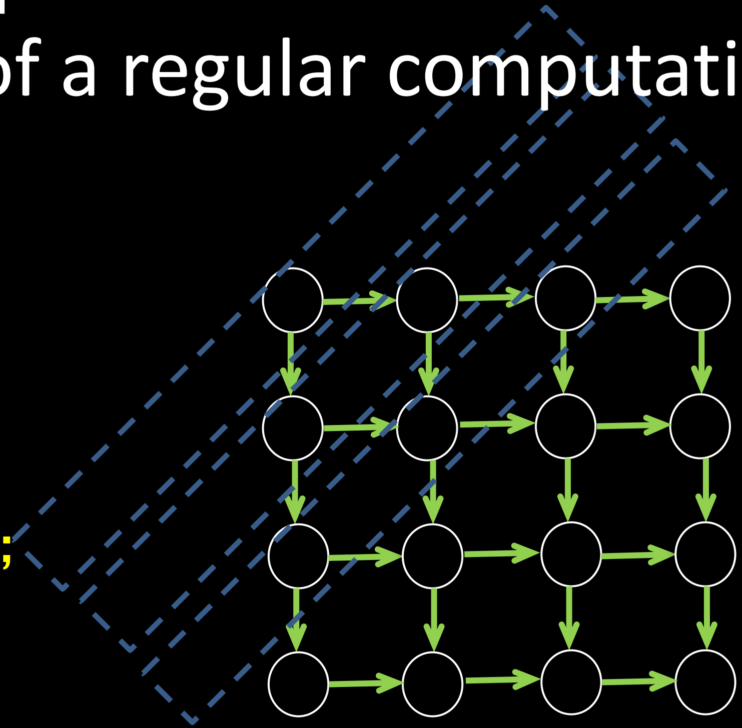


Example

Automatic transformation of a regular computation

- Find parallelism

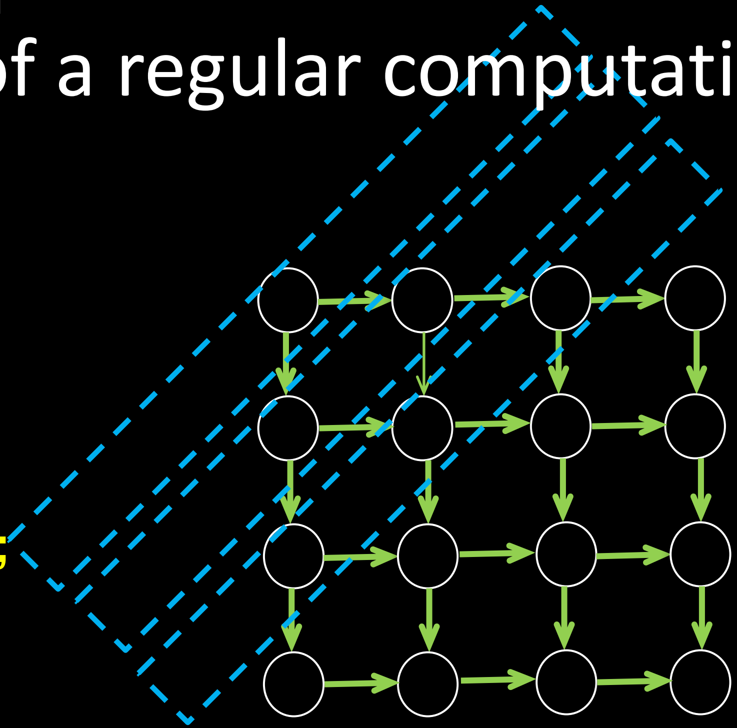
```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```



Example
Automatic transformation of a regular computation

- Transform the code

```
for (i=1; i<n; i++) {
 for (j=1; j<n; j++) {
   a[i][j]=a[i][j-1]+a[i-1][j];
}}
```



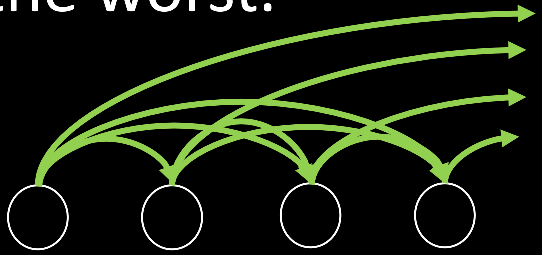
```
for (k=4; k<2*n; k++)  forall (i=max(2,k-n):min(n,k-2)) a[i][k-i]=...
```

Analyzing irregular computations

- Analysis of the following code is difficult and sometimes impossible

```
for (i=1; i<n; i++) {
    a[c[i]]=a[d[i]]+1;
}
```

- Dependences are a function of the values of c and d
- In the absence of additional information, the compiler must assume the worst.



- And this precludes automatic transformations (parallelization, vectorization, data distribution,…)

Irregularity is a beast programming language researchers have been fighting for a long time

It has defeated us sometimes

Ignoring the importance of irregular computations was one of the Reasons for the failure of High Performance Fortran

Taming the irregular beast

- Compilers and Runtime (Section 3)
- Better notations (Section 4)

3. COMPILER AND RUNTIME SOLUTIONS FOR LOW LEVEL IRREGULAR PROGRAMMING

Two approaches to dealing with irregular computations

- Analyzing irregular codes directly
  - Statically
  - Dynamically
- Converting into higher level of abstraction (dense array computations)

Static analysis

- In some cases, it is possible to analyze irregular algorithms written in conventional notation.
- More effort than for regular/linear subscript computations

Static analysis

```
do k = 1, n
    q = 0
    do i = 1, p
        if ( x(i) > 0 ) then
            q = q + 1
            ind(q) = i
        end if
    end do
    do j = 1, q
       x(ind(j)) = x(ind(j))* y(ind(j))
    end do
end do
```

Dependence analysis

Static analysis

```
do k = 1, n
    q = 0
    do i = 1, p
        if ( x(i) > 0 ) then
            q = q + 1
            ind(q) = i
        end if
    end do
    do j = 1, q
       x(ind(j)) = x(ind(j))* y(ind(j))
    end do
end do
```

Need information about **`ind(j)`**

Dependence analysis

Static analysis

```
do k = 1, n
    q = 0
    do i = 1, p
        if ( x(i) > 0 ) then
            q = q + 1
            ind(q) = i
        end if
    end do
    do j = 1, q
       x(ind(j)) = x(ind(j))* y(ind(j))
    end do
end do
```

Need information about **`ind(j)`**

Can be found by analyzing the program

Dependence analysis

Static analysis

```
do i=1, n
    do j=2, iblen(i)
        do k=1, j-1
            x(pptr(i)+k-1) = ...
        end do
    end do
    do j=1, iblen(i)-1
        do k=1, j
            ... = x(iblen(i)+pptr(i)+k-j-1)
        end do
    end do
end do
```

Array privatization

Static analysis

```
do i=1, n
    do j=2, iblen(i)
        do k=1, j-1
            x(pptr(i)+k-1) = ...
        end do
    end do
    do j=1, iblen(i)-1
        do k=1, j
            ... = x(iblen(i)+pptr(i)+k-j-1)
        end do
    end do
end do
```

To decide if **x** can be privatized

Array privatization

Static analysis

```
do i=1, n
    do j=2, iblen(i)
        do k=1, j-1
            x(pptr(i)+k-1) = ...
        end do
    end do
    do j=1, iblen(i)-1
        do k=1, j
            ... = x(iblen(i)+pptr(i)+k-j-1)
        end do
    end do
end do
```

To decide if **x** can be privatized
Is sufficient to show that a write precedes each read

Array privatization

Static analysis

- A possible approach is to query the control flow graph, bounding the search.

Yuan Lin and David Padua. Compiler analysis of irregular memory accesses. (PLDI '00).

Dynamic analysis

- Embedded dependence analysis
- Inspector executor
- Speculation

Embedded dependence analysis

- This is a clever mechanism due to Zhu and Yew.

```
for i=1; i<n; i++
      a(k(i)) = c(i) + 1
```

$\Rightarrow$

```
a_tag(k(:)) = 0
parallel for i=1;i<n;i++
     critical a(k(i))
          if a_tag(k(i)) < i
               a(k(i)) = c(i) + 1
               a_tag(k(i)) = i
```

Zhu, C. Q., & Yew, P. C. (1987). SCHEME TO ENFORCE DATA DEPENDENCE ON LARGE MULTIPROCESSOR SYSTEMS. IEEE Transactions on Software Engineering, SE-13(6), 726-739

Inspector executor

- Parallel schedule and communication aggregation computed at execution time
  - Analyzing the memory access pattern of a code segment (typically a loop)
- Overhead reduced when the code segment (loop) is executed multiple times with the same access pattern

See Encyclopedia of Parallel Computing
Run Time Parallelization (Saltz and Das)

Speculation

- There is an extensive body of literature on speculation.
- A code segment is executed in parallel in the hope that there is no problem.
- If there is, execution backtracks and the code segment is re-executed serially

See Lawrence Rauchwerger, David A. Padua: The LRPD Test: Speculative Run-Time Parallelization of Loops with Privatization and Reduction Parallelization. PLDI 1995: 218-232

Encyclopedia of Parallel Computing

Thread level speculation (Torrellas)

Transactional Memories (Herlihy)

Converting into dense array computations

- The objective is to convert sparse computations into equivalent dense form.
- This increases the number of operations involving zeroes (or identity)
  - Sometimes making the computation impossible
- But it allows the application of compile-time transformations.
- Sparsity is recovered by applying a final transformation (see below)

Converting into dense array computations

```
for(col=0; col<cols; col++) {
     for(row=0; row<dimensions; row++) {
          leftCell = left.Rows[row];
          while( leftCell != NULL ) {
               result[row][col] +=leftCell->Value
                                * right[leftCell->ColIndex][col];
               leftCell = leftCell->ColNext;
```

```
for( col = 0; col < cols; col++ )
     for( row = 0; row < dimensions; row++ )
        result[row][col] += A_Valuep[row][:] * right[:][col];
```

Converting into dense array computations

Harmen L. A. van der Spek, Harry A. G. Wijshoff:
Sublimation: Expanding Data Structures to Enable Data Instance Specific Optimizations.
LCPC 2010: 106-120

Anand Venkat, Mary Hall, and Michelle Strout. 2015. Loop and data transformations for sparse matrix code. In *Proceedings of the 36th ACM SIGPLAN Conference on Programming Language Design and Implementation*(PLDI '15).

Where do we stand?

- There is an extensive body of literature on compilation and runtime to facilitate the parallel programming of irregular computations.
- Although more ideas are important, what we need most urgently is an evaluation of their effectiveness.
  - A means for refinement and progress.
- All compiler techniques suffer from lack of understanding of their effectiveness.

Need tools to evaluate progress

- Measuring advances in compiler technology is crucial for progress.
- An idea: a publicly available repository
  - Containing extensive and representative code collections.
  - Keep track of results for compilers/techniques along time.
- Ongoing work on one such repository:
  - *LORE: A Loop Repository for the Evaluation of Compilers* by Z.i Chen, Z. Gong, J. Josef Szaday, D. Won, D. Padua, A. Nicolau , A. Veidenbaum , N. Watkinson , Z. Sura, S. Maleki, J. Torrellas, G. DeJong**. IISWC-2017**

4. NOTATIONS

4. NOTATIONS

4.1 ABSTRACTION

What are Programming abstractions ?

- An abstraction is formed by reducing information.
- In everyday life, it forms the world of ideas where we can reason. No (natural language) statement can be made without abstractions

What are Programming abstractions ?

- Abstract *machine language/lower level* implementation
  - Scalar computations:
    - `a=b*c+d^3.4`
    - No loads, stores, register allocation.
  - Scalar loops:
    - `for (i= ….) {…}`
    - No if statements, branch/goto instructions.
    - Gives structure to the program.
- Abstract algorithm implementation
    - `min(A(1:n))`
    - `it = find (myvec.begin(), myvec.end(), 30);`
    - Hide algorithm, data representation

What are the benefit of using abstractions?

- Programmer productivity (expressiveness)
  - Codes are easier to write
  - To understand, maintain
- Portability
- Optimization (manual and automatic)
  - Programs are easier to analyze
  - Easier to transform
  - Deeper transformations are enabled

Abstractions and irregular computations

- What abstractions should we use for programming irregular computations at a higher level?
- Best answer seems to be to represent irregular algorithms in terms of:

  Dense array operations

4. NOTATIONS

4.2. ARRAY NOTATION AND IRREGULAR COMPUTATIONS

Why array notation

- Could access arrays in scalar mode
- But operations on aggregates are (often) easier to read.

```
for (i=0; i<n; i++) for (j=0;j<n;j++)
      a[i][j]=b[i][j]+c[i][j]
```

```
a=b+c
```

- They are well understood abstractions.
- Powerful.

Array operations and performance

- But usefulness of array notation go beyond expressiveness.
- There are at least three reasons to use array abstractions for performance
  - To represent parallelism
  - To reduce the overhead of interpretation.
  - To enable automatic optimizations.

Representation of parallelism

- Popular in the early days:

```
    do 10 i = 1, 100, 2
       M (i) = .true.
       M (i+1) = .false.
10 continue
   A(*) = B(*) + A(*)
   C(M(*)) = B(M(*)) + A(M(*))
```

Illiac IV Fortran

- Used today but not widely

```
c[i][ j] = __sec_reduce_add( a[ i][:] * b[:][j]);
```

Cilk plus

- Tensor flow
- Futhar*k*
  - T. Henriksen, N. Serup, M. Elsman, F. Henglein, and C. Oancea. *Futhark: Purely Functional GPU-Programming with Nested Parallelism and In-Place Array Updates* PLDI 2017

Array operations and optimizations An example

- Applying arithmetic rules such as associativity of matrix-matrix multiplication to reduce the number of operations .
  - Which one is better?
    - `(((A*B)*C)*D)*E`,
    - `(A*((B*C)*D))*E`,
    - `(A*B)*((C*D)*E)`,
    - …
  - Find best using dynamic programming
  - *Yoichi Muraoka and David J. Kuck. 1973. On the time required for a sequence of matrix products. Commun. ACM 16, 1 (January 1973), 22-26.*

Reduce overhead of interpretation

- See Haichuan Wang, David A. Padua, Peng Wu: Vectorization of apply to reduce interpretation overhead of R. OOPSLA 2015: 400-415

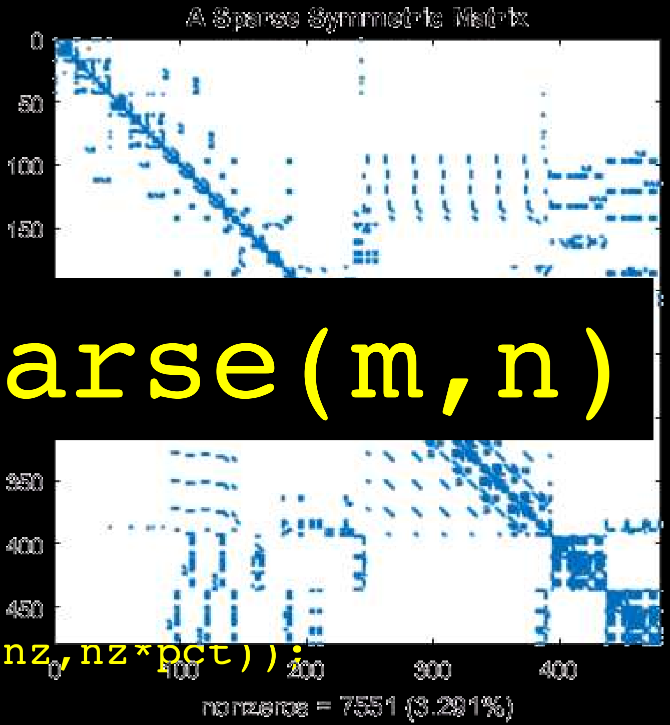
Array operations and irregular computations

- Array operations are an ideal notation to manipulate sparse arrays.

Sparse arrays in MATLAB

```
load west0479.mat
A = west0479;
S = A * A' + speye(size(A));
pct = 100 / numel(A);

figure
spy(S)
title('A Sparse Symmetric Matrix')
nz = nnz(S);
xlabel(sprintf('nonzeros = %d (%.3f%%)',nz,nz*pct));
```



```
S = sparse(m,n)
```

- Arrays appear as dense, but represented internally as spaarse.
- The interpreter and libraries handle the sparseness

Sparse arrays can be abstracted as dense arrays in compiled languages

- Bik and Wijshoff* developed a strategy to translate dense array programs onto sparse equivalents.

```
do i=1,n
    acc=acc+a(i,:)*<1:n>
```

```
do i=1,n
   do j=1,n
      if (i,j) ∈ E_a then
         acc=acc+a'(σ_a(i,j))*j
```

```
do i=1,n
   do ad∈PAD_a
      j=π_2σ_a^{-1}(ad)
      acc=acc+a'(ad)*j
```

```
do i=1,n
   do ad=alow(i),ahigh(j)
      j=aind(ad)
      acc=acc+aval(ad)*j
```

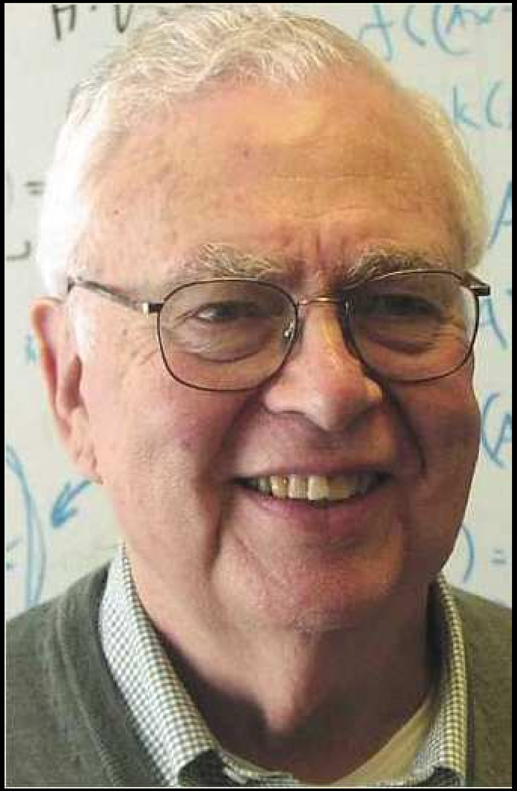
*Aart J. C. Bik, Harry A. G. Wijshoff: Compilation Techniques for Sparse Matrix Computations. International Conference on Supercomputing 1993: 416-424

Array operations and irregular computations

- Not only can array notation be used to represent dense computations, it can be used for a number of other domains.
- A wide range of domains

How wide?

- At the time (ca. 1959), Illinois, with the first of its ‘ILLIAC’ supercomputers, was a great centre of computing, and Golub showed his affection for his Alma Mater by endowing a chair there 50 years later. Rumour has it that the funds for the gift came from Google stock acquired in exchange for some advice on linear algebra. Google’s PageRank search technology starts from a matrix computation — an eigenvalue problem with dimensions in the billions. Hardly surprising, Golub would have said: everything is linear algebra. Nature 2007



New operators
Arrays for graphs algorithms

- A simple algorithm for SSSP (Bellman-Ford) can be represented as follows:

```
for(i=1; i<=n; i++)
    $d = d \oplus A \otimes d$
```

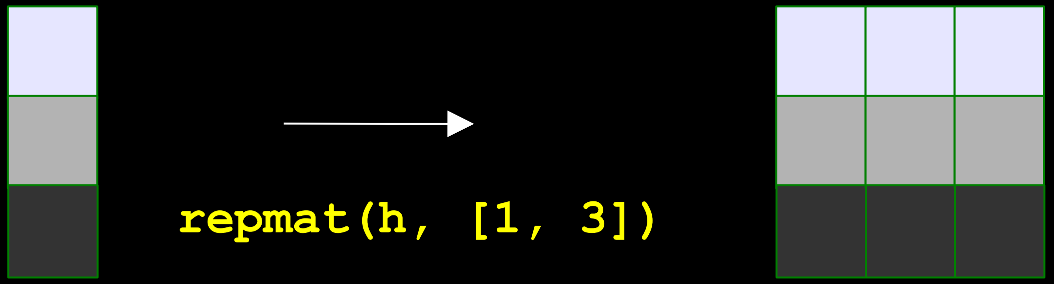
Jeremy Kepner and John Gilbert. 2011. Graph Algorithms in the Language of Linear Algebra. SIAM Press

T. Mattson, D. Bader, J. Berry, A. Buluc, J. Dongarra,C. Faloutsos, J. Feo, J. Gilbert, J. Gonzalez, B.Hendrickson, J. Kepner, C. Leiserson, A. Lumsdaine, D. Padua, S. Poole, Steve Reinhardt, M. Stonebraker, S. Wallach, A. Yoo. Standards for Graph Algorithm Primitives

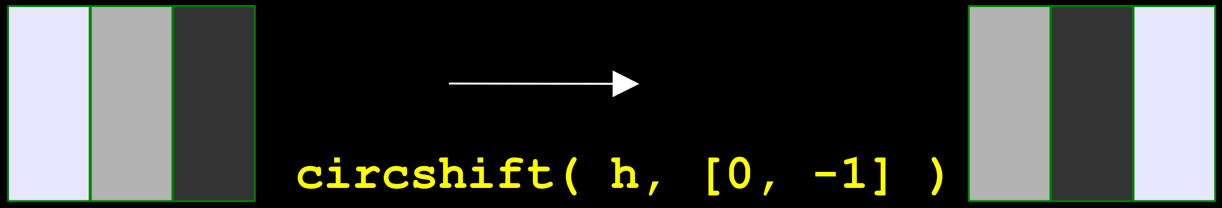
4. NOTATIONS

4.3. EXTENSIONS TO ARRAY NOTATION FOR PARALLELISM

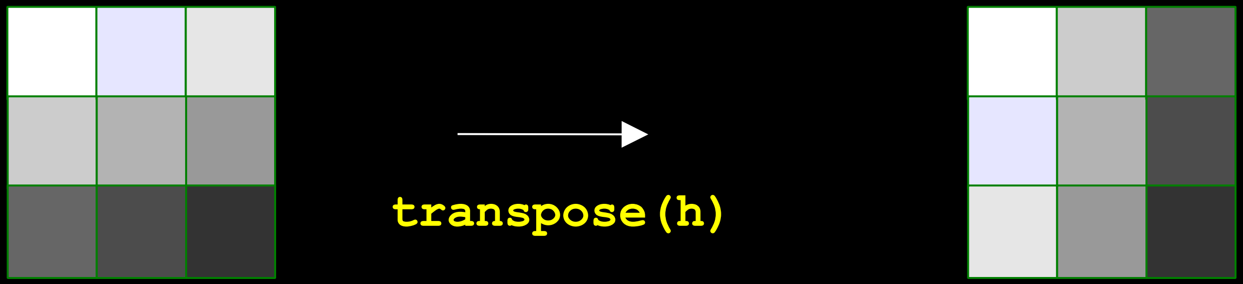
Communication as array operations



```
repmat(h, [1, 3])
```

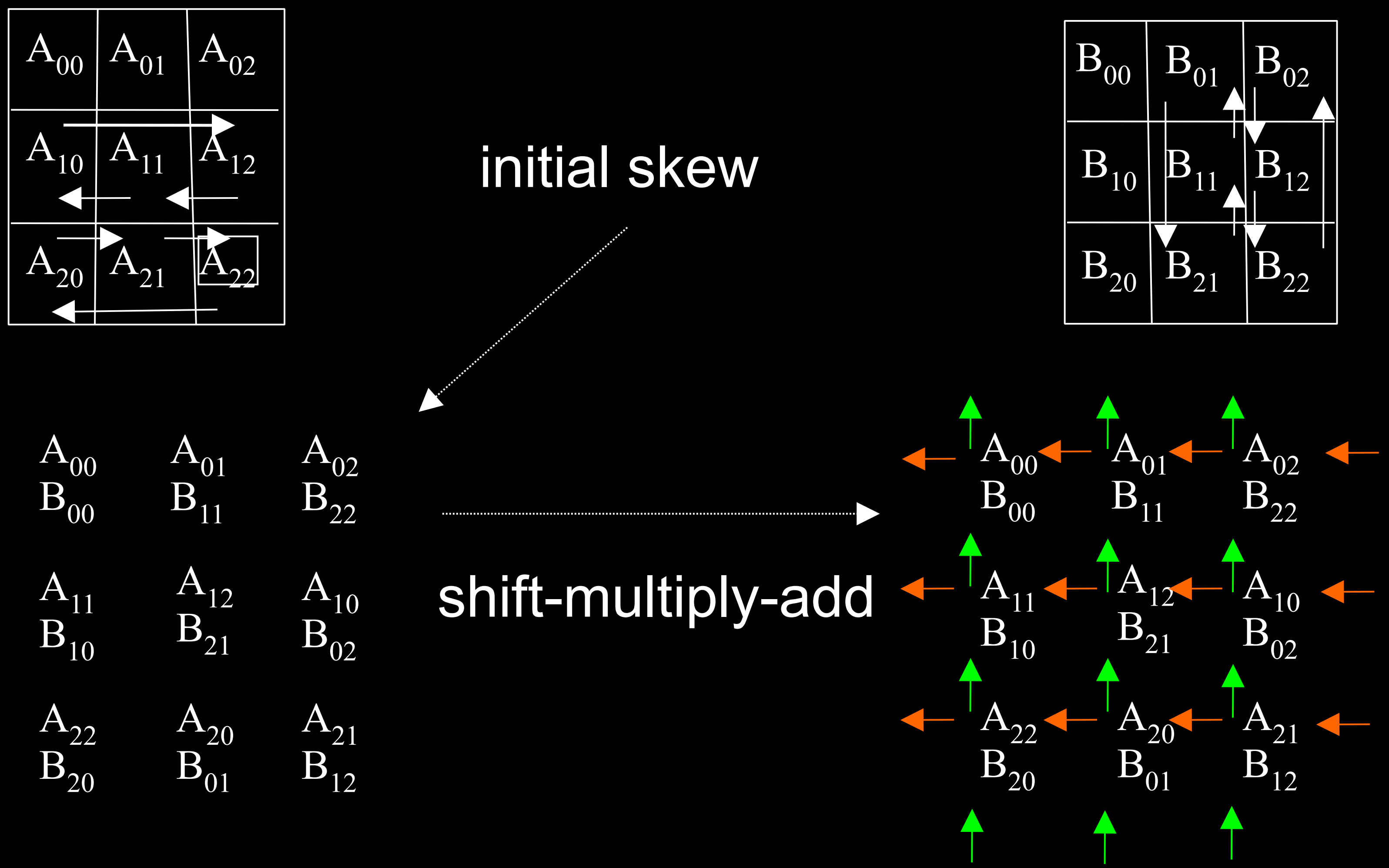


```
circshift( h, [0, -1] )
```



```
transpose(h)
```

Cannon’s algorithm



A_{00} A_{01} A_{02}
A_{10} A_{11} A_{12}
A_{20} A_{21} A_{22}
B_{00} B_{01} B_{02}
B_{10} B_{11} B_{12}
B_{20} B_{21} B_{22}
initial skew
A_{00} B_{00} A_{01} B_{11} A_{02} B_{22}
A_{11} B_{10} A_{12} B_{21} A_{10} B_{02}
A_{22} B_{20} A_{20} B_{01} A_{21} B_{12}
shift-multiply-add
A_{00} B_{00} A_{01} B_{11} A_{02} B_{22}
A_{11} B_{10} A_{12} B_{21} A_{10} B_{02}
A_{22} B_{20} A_{20} B_{01} A_{21} B_{12}

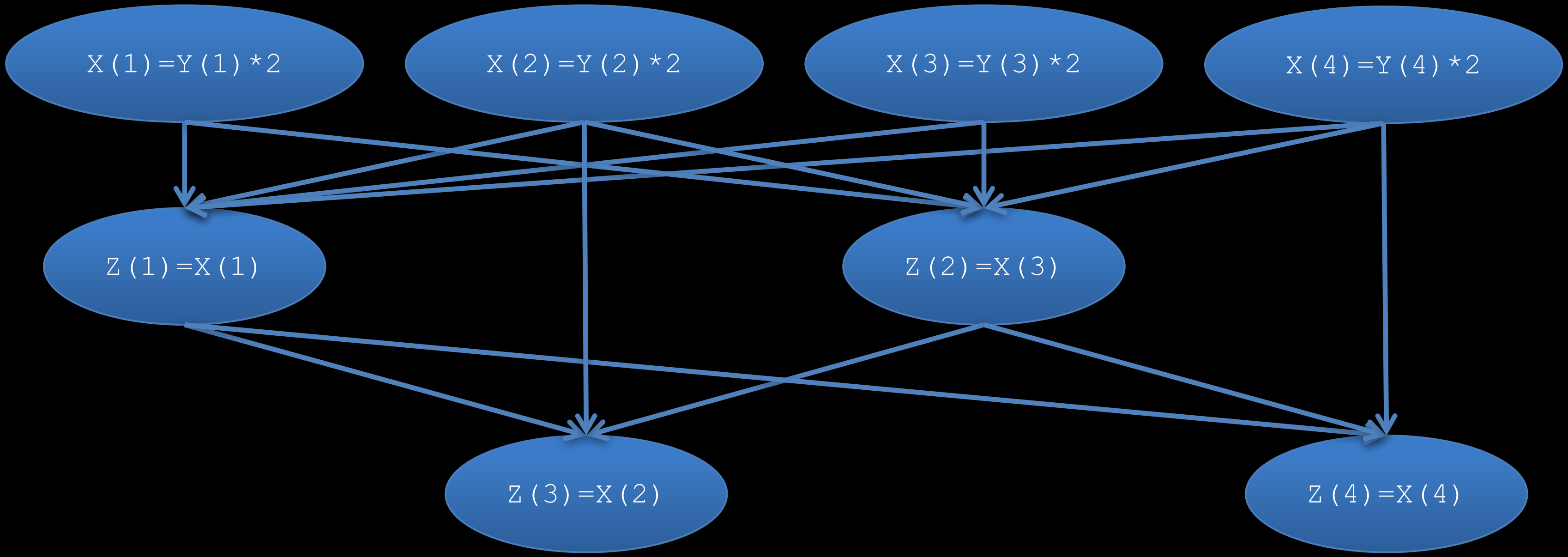
Cannnon’s Algorithm

```
for k=1:n
    c = c + a × b;
    a = circshift(a,[0 -1]);
    b = circshift(b,[-1 0]);
end
```

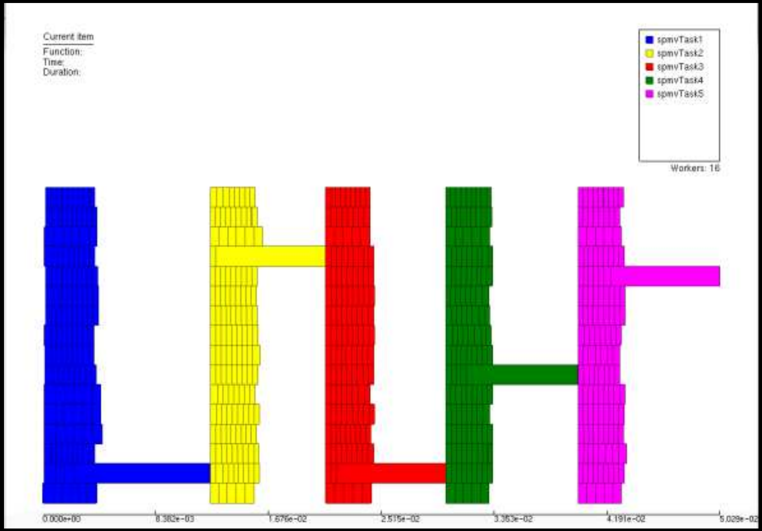
James C. Brodman, G. Carl Evans, Murat Manguoglu, Ahmed H. Sameh, María Jesús Garzarán, David A. Padua: A Parallel Numerical Solver Using Hierarchically Tiled Arrays. LCPC 2010: 46-61

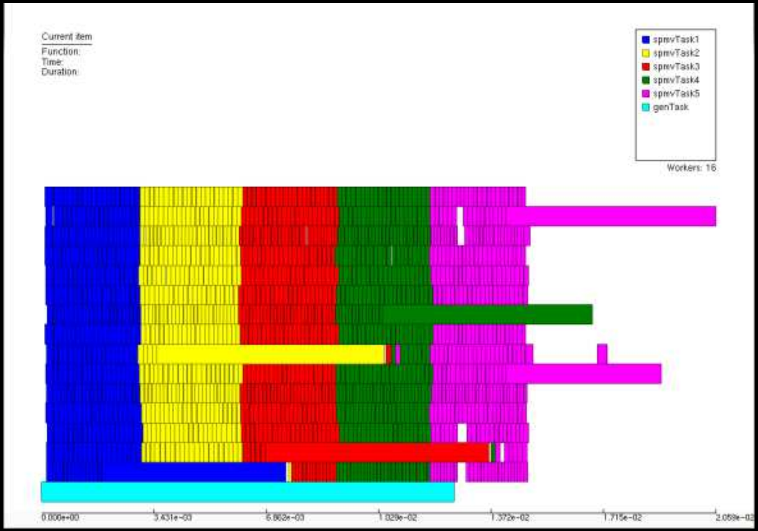
Barrier elimination

```
X(1:4)=Y(1:4)*2;
Z(1:2)=X([1,3]);
Z(3:4)=X([2,4]);
```



Barrier elimination





Asynchronous semantics

- In some cases, it could be desirable to delay updates across the whole machine. At some point, a global update is made.

- This is a particular example of communication in asynchronous algorithms where data assignment suffer delays.

Tiled Linear Algebra – SSSP Example

```
// the distance vector
d = inf; d(1) = 0;
// the mask vector
L = 0; L(1) = 1;
while (L.nnz())
  for i = 1:LOC_ITER
    r <- d + (trans(A)*(d*.L));
    L <- (r != d);
    d <- r;
 r += d;
  L <- (r != d);
  d <- r;
```

Partial multiplication

Global reduction

Local computation

Saeed Maleki, G. Carl Evans, David A. Padua:
Tiled Linear Algebra a System for Parallel Graph Algorithms. LCPC 2014: 116-130

4. NOTATIONS

4.3. WHERE DO WE STAND?

Array notation

- Array notation has a long history
- Is a powerful notation that can be used for a wide range of applications.
- For parallelism, it is an unfulfilled promise
- The highly mathematical notation is a challenge
- We see increase use of the notation and expect it to become popular also for irregular algorithms.
- Challenges:
  - develop the notation
  - Design compiler optimizations

5. EPILOG

- In 1976 not much was known about parallel programming of irregular computations.
- 40 years later if is impressive how much have been learned on compilation and notations.
- But the development of widely used tools based on these ideas remain a challenge.